Phenomenology from Lattice QCD

The Physics of Beauty and Strangeness

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1. Introduction

2. Lattice QCD and the Unitarity Triangle

3* Inclusive Quantities in B-Physics

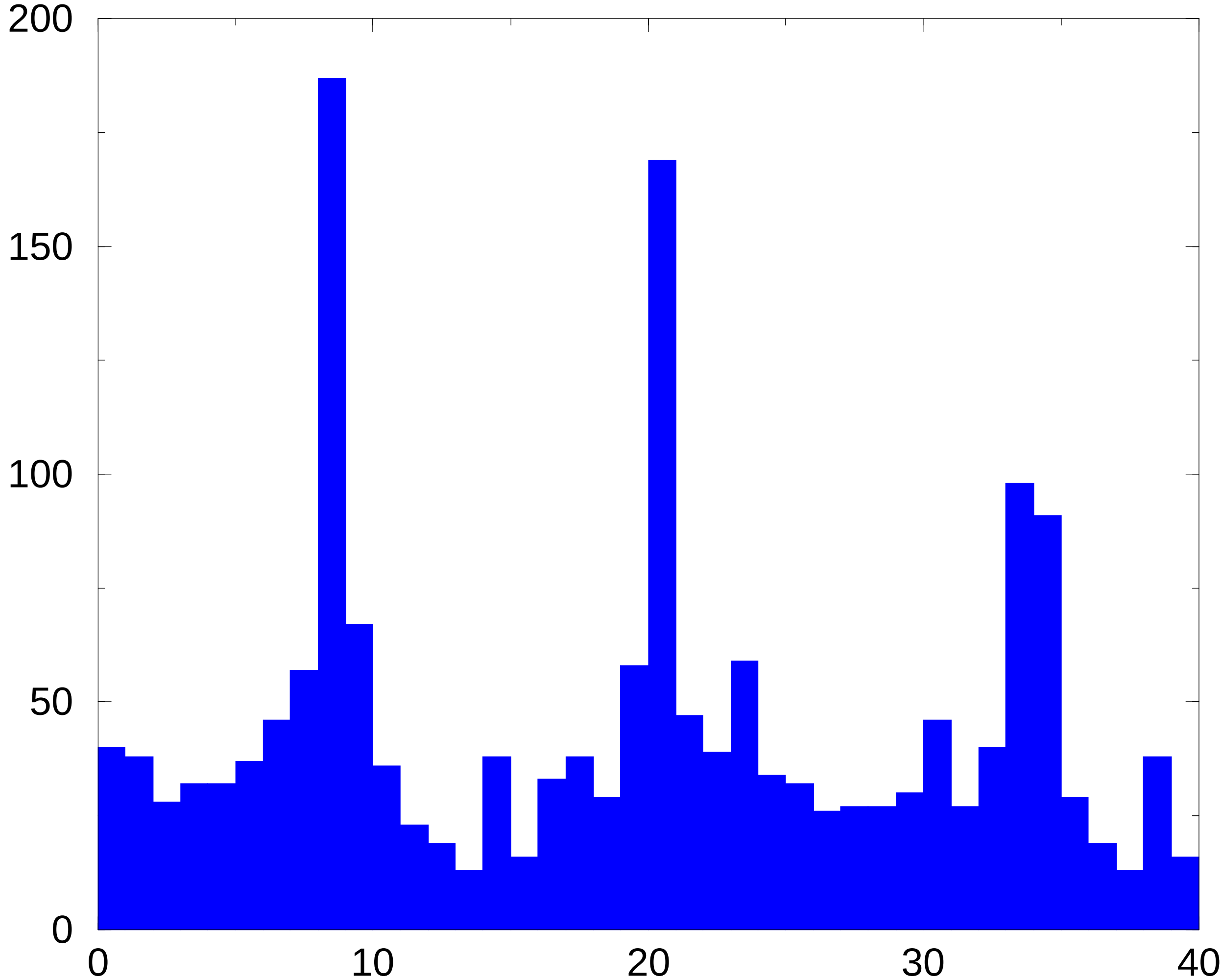
4* m_s and m_b

5 $K \to \pi\pi$ Decays

6 Conclusions

*The slides for these items are included, but they will not be discussed in the lecture itself.

1. Introduction



Number of papers per month submitted to **hep-lat** since January 1998

Introduction - Cont.

Why?

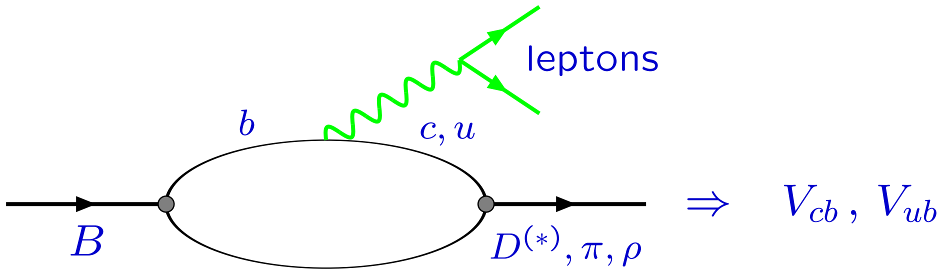
- The primary rôle of lattice QCD and large scale numerical simulations is to enable the evaluation of non-perturbative QCD effects in physical amplitudes.

- For the topics discussed in this talk, it is frequently our inability to quantify the long-distance QCD effects in weak processes which is the dominant source of uncertainty in determining fundamental quantities from experimental measurements.

- **How Precisely?** Systematic errors will be outlined briefly with the results. Increasingly calculations are being performed with 2 flavours of sea quarks, but it should be remembered that the masses of the sea and valence quarks are large (m_π/m_ρ is typically about 0.6 or more) so that extrapolations are needed.

- **What?** The material presented here represents only a small fraction of lattice results in general and lattice phenomenology in particular.
  See: Proceedings of the Annual Lattice Conferences;
  Report of the ECFA panel on the Requirements for High Performance Computing for Lattice QCD, CERN 2000-002.

Exclusive Semi-Leptonic B-Decays



Lorentz + Parity Invariance $\Rightarrow$ it is convenient to express the amplitudes in terms of invariant form-factors:

$$\langle\, P(p_P)\,|V_\mu(0)\,|\,B(p_B)\,\rangle \;=\; f^0(q^2)\,\frac{M_B^2-M_P^2}{q^2}q_\mu + f^+(q^2)\left[(p_B+p_P)_\mu - \frac{M_B^2-M_P^2}{q^2}q_\mu\right]$$

- $q \equiv p_B - p_P$.
- Parity invariance $\Rightarrow$ only V (from V-A) contributes when the final-state hadron is a pseudoscalar.
- For $B \to$ vector decays, both the V and A currents contribute and there are four form-factors $(A_{1,2}, A(= A_0\text{-}A_3), V)$.

$B \to D^{(*)}\ell\nu$-Decays

- Lattice calculations could (in principle and perhaps in practice) make a contribution to the determination of V_{cb} by evaluating QCD corrections to the form factors.

- This is a challenging task, since, to make an impact, one needs to calculate small corrections to the heavy quark limit.

$$\frac{d\Gamma(B \to D^* l\nu)}{d\omega} = \text{Kinematic Factor} \times |V_{cb}|^2\, \mathcal{F}^2(\omega)\ ,$$

  where

$$\mathcal{F}(1) = 1 + \text{corrections.}$$

  To make an impact one has to be able to calculate the $1/m_Q^2$ non-perturbative corrections.

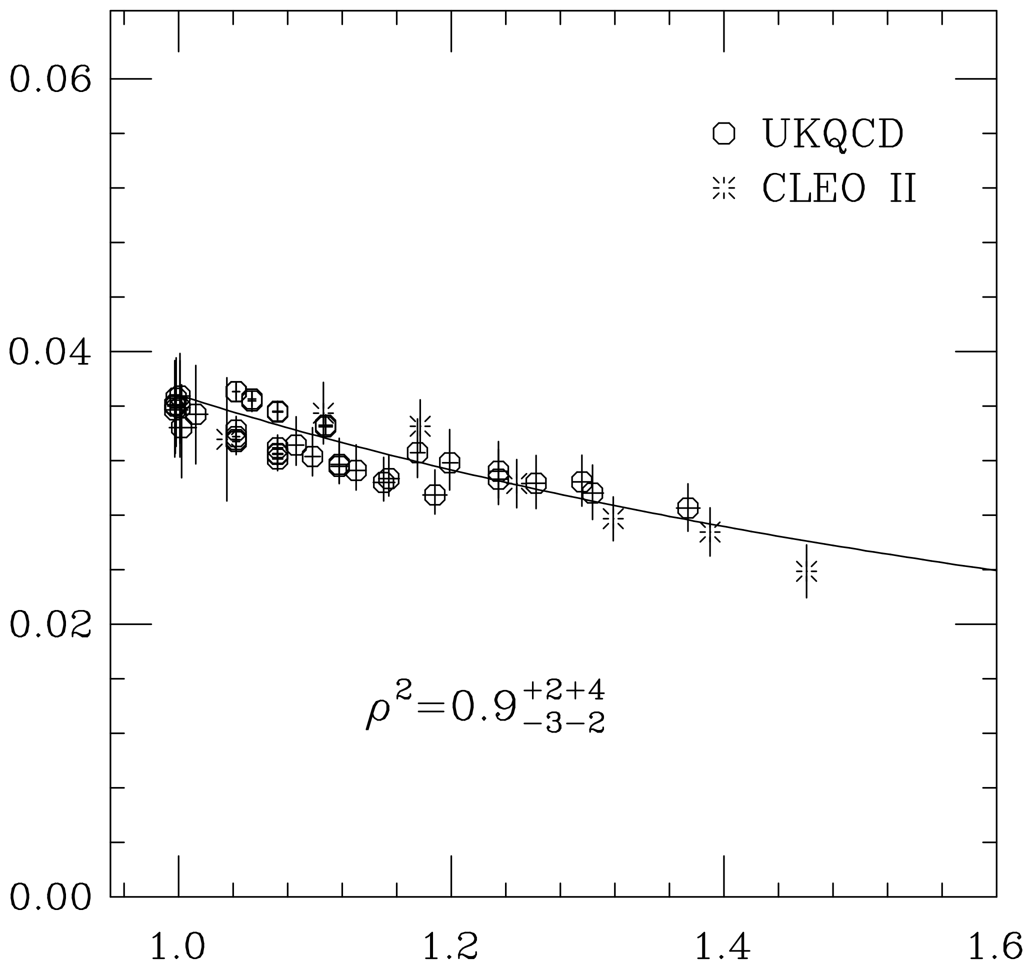
- In 1999 Hashimoto et al. suggested (and verified) that such a calculation may be possible, using ratios of matrix elements (in which there is a significant cancellation of statistical and perhaps systematic errors) e.g.

$$\frac{\langle\, D\,|\,\bar c\gamma_0 b\,|\,\bar B\,\rangle\langle\,\bar B\,|\,\bar b\gamma_0 c\,|\,D\,\rangle}{\langle\, D\,|\,\bar c\gamma_0 c\,|\,D\,\rangle\,\langle\,\bar B\,|\,\bar b\gamma_0 b\,|\bar B\,\rangle} = |h_+(1)|^2\ .$$

  Need excellent control of the systematic errors!

- The ω distribution has been successfully computed in the past in lattice simulations. fig

$B \to D^{(*)}\ell\nu$-Decays - Cont.



Fit of the UKQCD lattice results for $|V_{cb}|\mathcal{F}(\omega)$ (1995) to the experimental data from the CLEO collaboration.

$B \to \pi, \rho$ Semileptonic Decays

- There has been more effort recently in lattice computations of heavy→light exclusive semileptonic decays ⇒ V_{ub}.

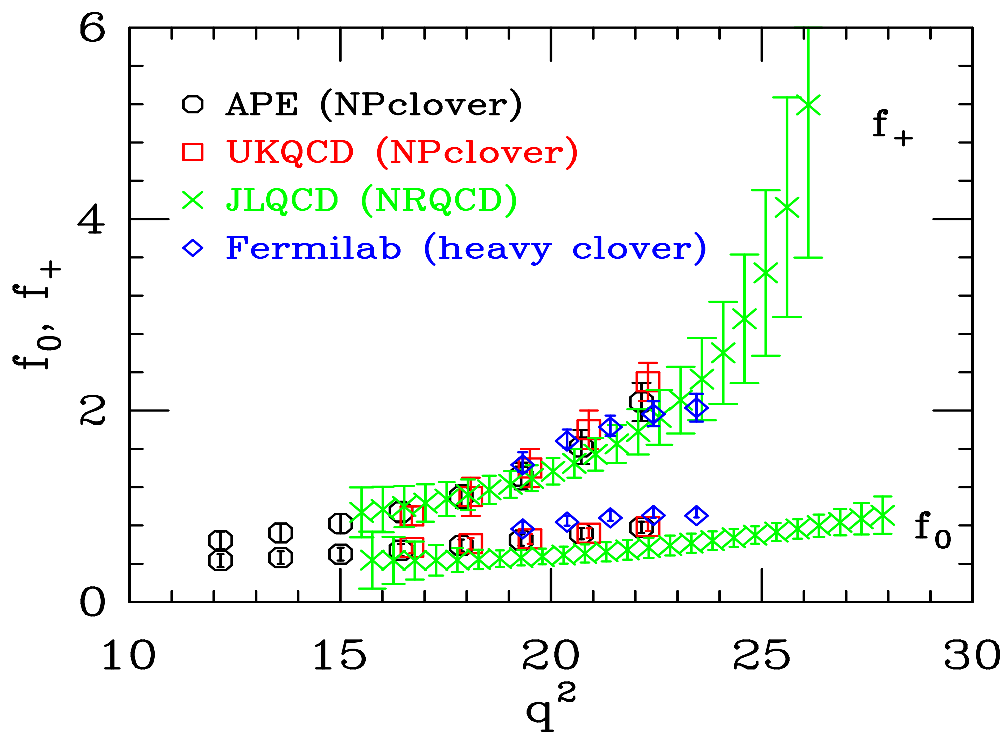
- The simulations for $B \to \pi, \rho$ semileptonic decays require $p_{\pi,\rho}$ to be small ⇒ only obtain results at large value of q^2.

- Calculations have been performed for many years now. At Lattice 2000 updated results were presented from the APE, UKQCD, JLQCD and Fermilab Collaborations.
  "Results were fairly consistent in the region where all groups had direct calculations
  ($19\,\text{GeV}^2 < q^2 < 23\,\text{GeV}^2$)." fig

- Much effort is being devoted to extrapolating the results to smaller values of q^2, using as many theoretical constraints as possible (HQET, unitarity and analyticity, kinematical constraints, soft pion relations $\cdots$).

- The most meaningful applications of lattice results are/would be to the experimental distibutions at large values of q^2. fig

$B \to \pi \ell \nu$-Decays



Recent data for $B \to \pi \ell \nu$ formfactors f_+ and f_0. Only statistical errors are shown.

From C.Bernard - Latt00

$$\langle \pi(\vec{k}\,) \,|\, V_\mu \,|\, B(\vec{p}\,)\rangle = f_+(q^2)\Big[p_\mu + k_\mu - \frac{(M_B^2 - M_\pi^2)q_\mu}{q^2}\Big] + f_0(q^2)\frac{(M_B^2 - M_\pi^2)q_\mu}{q^2}$$

where $q = p - k$.

$B \to \pi\,,\rho$ Semileptonic Decays

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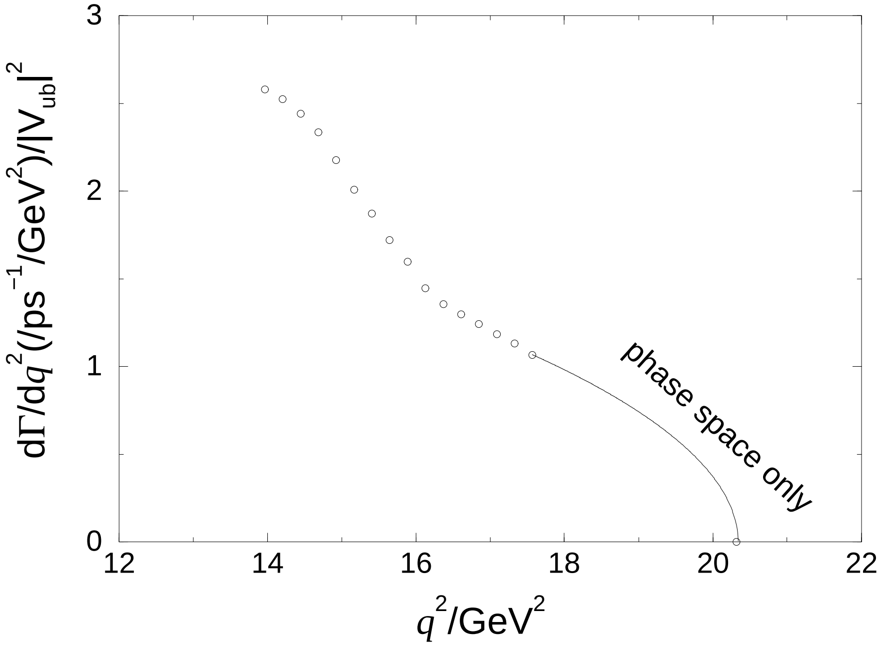


UKQCD results for the distribution for $b \to \rho$ decays in the range 14 GeV$^2 < q^2 < 20.3$ GeV^2, which corresponds to the high q^2 bin of CLEO.

- The region marked *phase space only* is inaccessible to the lattice calculations and the curve corresponds to taking the form-factors from the last lattice point and extrapolating using the phase-space only.

- Integrating the results in this bin the comparison takes the form:

$$\frac{\Delta\Gamma(14 < q^2/\text{GeV}^2 < 20.3)}{\text{ps}^{-1}\,\text{GeV}^{-2}} = 8.3\,|V_{ub}|^2$$

(UKQCD preliminary)

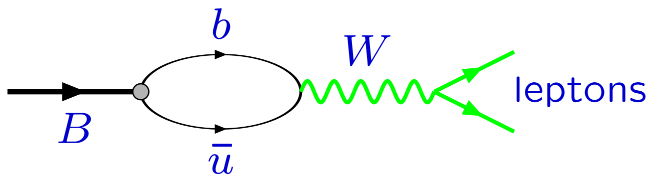
$$= 7.1(2.4) \times 10^{-5}$$

(CLEO)

from which one obtains $V_{ub} = 2.9(0.5) \times 10^{-3}$.

The Decay Constants f_B and f_{B_s}

f_B :



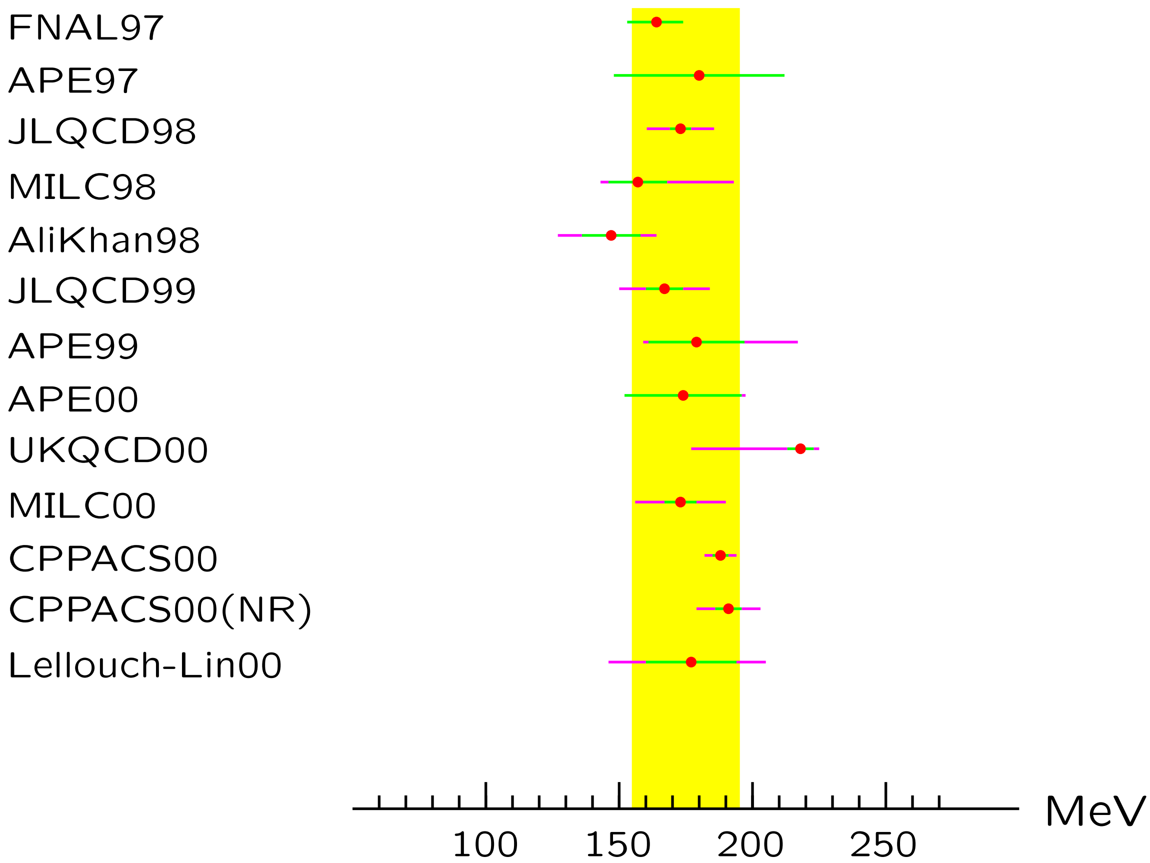
The relevant matrix element is

$$\langle\, 0 \,|\, A_\mu(0) \,|\, B(p)\, \rangle = i f_B p_\mu$$

- Lorentz and Parity Invariance $\Rightarrow$ all QCD effects parametrized in terms of a single number f_B.
  Normalization used here corresponds to $f_\pi \simeq 132$ MeV.

- Quenched calculations have been performed for about 15 years now.
  Careful analysis of all systematic errors (apart from quenching) is possible
  (see e.g. C.Bernard, Latt00 Proceedings). fig

- The emphasis is turning now to unquenched calculations. There is some belief that $f_{B,N_f=2}$ is 10-15% larger that the decay constant in the quenched approximation. fig

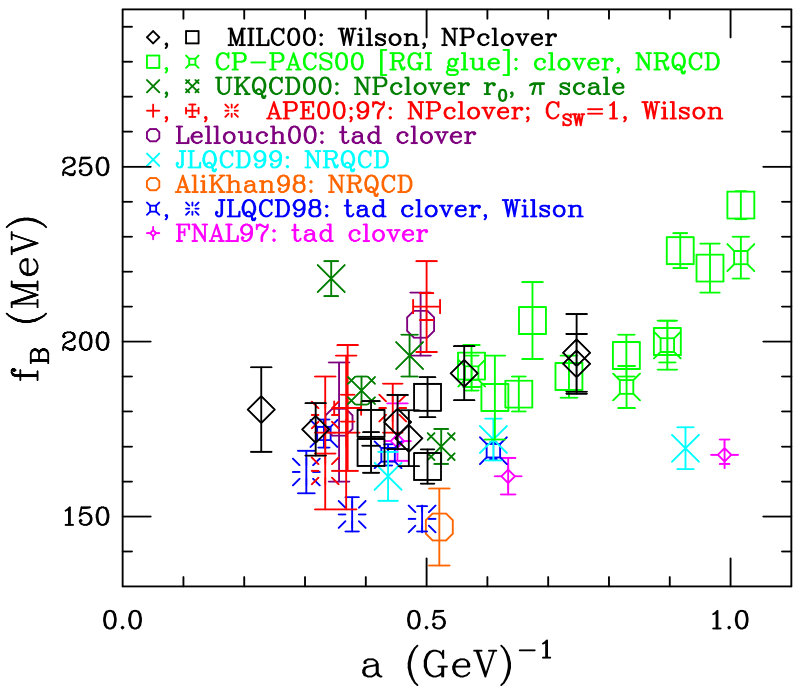


Results for f_B from various groups in the quenched approximation. Green lines indicate statistical errors and purple lines show statistical and systematic errors combined in quadrature.

C. Bernard, (Heavy Quark Physics Rapporteur at Lattice 2000):

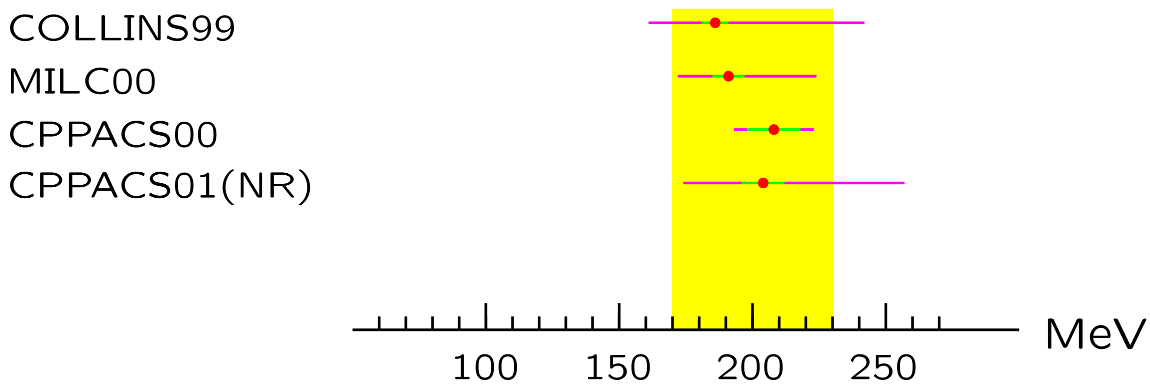
- $$f_{B,\text{quenched}} = 175 \pm 20\,\text{MeV} \ .$$

- An important source of systematic error is the that due to the finite lattice spacing. In quenched calculations this has been studied in considerable detail.



Recent world data for f_B in the quenched approximation as a function of the lattice spacing. Only statistical errors are shown.

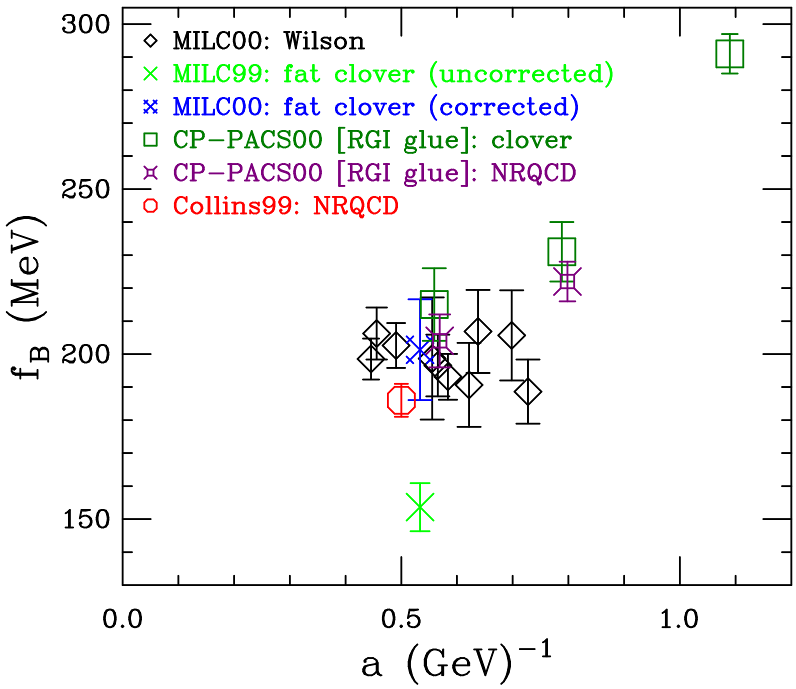
From C.Bernard - Latt00



Results for f_B from various groups for two flavours of sea quarks. Green lines indicate statistical errors and purple lines show statistical and systematic errors combined in quadrature.

C. Bernard, (Heavy Quark Physics Rapporteur at Lattice 2000):

- $$f_B = 200 \pm 30\,\text{MeV} \ .$$



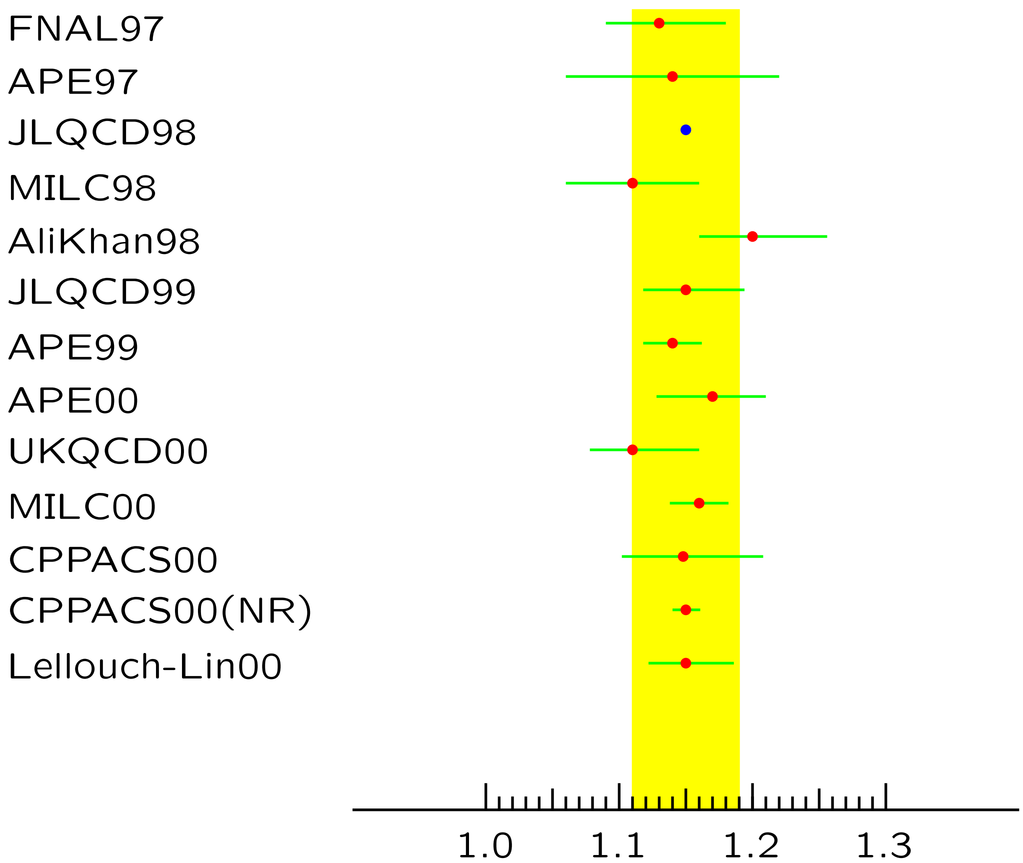
World data for f_B with $N_f = 2$ as a function of the lattice spacing. Only statistical errors are shown.

From C.Bernard - Latt00

- An important parameter in Unitarity Triangle analyses is

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} \ .$$

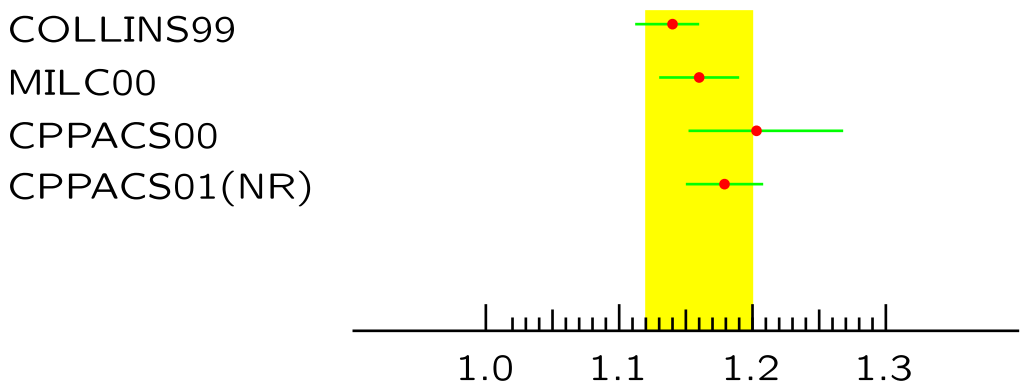
Lattice simulations indicate that the B-parameter varies slowly with the light-quark mass (see below), so it is interesting to present f_{B_s}/f_{B_d}. figs



Results for f_{B_s}/f_B from various groups in the quenched approximation. Statistical errors and systematic errors have been combined in quadrature.

C. Bernard, (Heavy Quark Physics Rapporteur at Lattice 2000):

- $$\left(\frac{f_{B_s}}{f_B}\right)_{\text{quenched}} = 1.15 \pm 0.04 \; .$$



Results for f_{B_s}/f_B from various groups with two flavours of sea-quarks. Statistical errors and systematic errors have been combined in quadrature.

C. Bernard, (Heavy Quark Physics Rapporteur at Lattice 2000):

- $$\frac{f_{B_s}}{f_B} = 1.16 \pm 0.04\ .$$

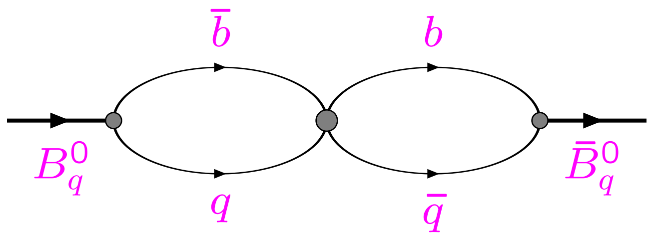
- The key point to note is that for both ξ and f_{B_s}/f_{B_d} it is the difference from 1 which is being computed. Lattice errors of 30% (which are conservative) therefore correspond to errors of only about 5% on ξ.

B-$\bar{B}$ Mixing

B_B: The relevant matrix element is:

$$M(\mu) \equiv \langle\, \bar{B}_q^0 \,|\, \bar{b}\gamma_\mu(1-\gamma_5)q \;\bar{b}\gamma^\mu(1-\gamma_5)q \,|\, B_q^0 \,\rangle\,,$$

where q represents d or s.



- It is convenient to define B-parameters:

$$M(\mu) = \frac{8}{3} f_{B_q}^2 m_{B_q}^2 B_{B_q}(\mu).$$

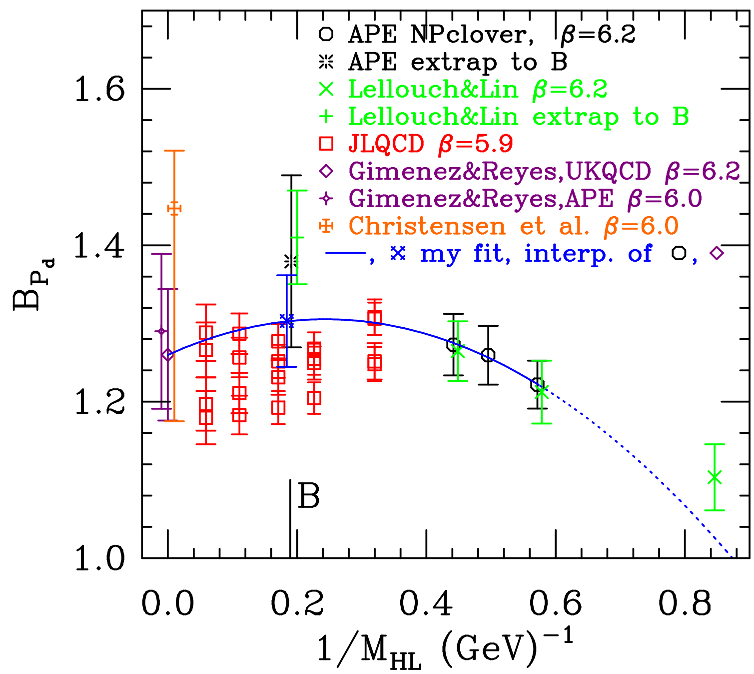
- The diagram can be viewed as representing the three-point correlation function on the lattice. All the quark propagators begin or end at the $\Delta B = 2$ operator which can be taken to be at the origin.

- Each $B_{B_q}(\mu)$ is scheme and scale-dependent, so it is convenient to define scheme-independent (up to NLO) quantity

$$\widehat{B}_{B_q}^{nlo} = \alpha_s(\mu)^{2/\beta_0}\left[1 + \frac{\alpha_s(\mu)}{4\pi} J_{n_f}\right]\; B_{B_q}(\mu).$$

  where J_{n_f} is a known constant.

B_B



Selection of recent data for quenched $\hat{B}_{P_d}$ as a function of $1/M_{P_d}$. Only statistical errors are shown (except for static-light results). From C.Bernard - Latt00

C. Bernard, (Heavy Quark Physics Rapporteur at Lattice 2000):

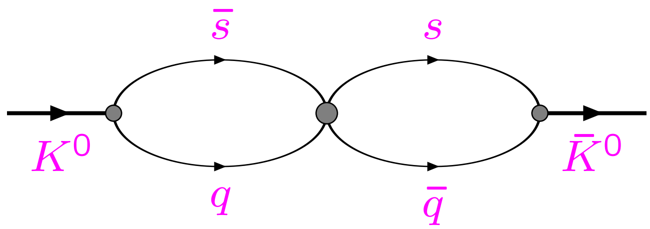
$$\hat{B}_{B_d} = 1.30 \pm 0.12 \pm 0.13$$

$$f_{B_d}\sqrt{\hat{B}_{B_d}} = 230 \pm 40\,\text{MeV}$$

$$\frac{\hat{B}_{B_s}}{\hat{B}_{B_d}} = 1.00 \pm 0.04$$

$$\xi = 1.16 \pm 0.05$$

K^0-$\bar{K}^0$ Mixing and B_K



The calculation is similar to that for B_B above. We need the matrix element:

$$\langle\, \bar{K}^0 \,|\, O^{\Delta S=2}_{V-A,V-A} \,|\, K^0 \,\rangle \ ,$$

where

$$O^{\Delta S=2}_{V-A,V-A} = \bar{s}\gamma_\mu(1-\gamma_5)d\,\bar{s}\gamma_\mu(1-\gamma_5)d \ .$$

Chiral symmetry plays a central rôle in the determination of B_K:

- This is the *flagship* calculation for staggered (KS) fermions.

- For simulations with Wilson-like fermions $O^{\Delta S=2}_{V-A,V-A}$ mixes with four other operators of dimension 6 $\Rightarrow$ subtractions are necessary $\Rightarrow$ loss of precision.

  There are two (related) proposals to circumvent this, based on Bernard's observation that CPS-symmetry (S=$s \leftrightarrow d$) implies that the parity-odd component, $O^{\Delta S=2}_{VA}$, renormalizes multiplicatively. (However, it is the parity-even component $O^{\Delta S=2}_{VV} + O^{\Delta S=2}_{AA}$ which we need.)

  1) *Twisted-Mass QCD:*

$$\mathcal{L} = \bar{\psi}(D_W + m_0 + i\mu_0\gamma_5\tau^3)\,\psi + \bar{s}(D_W + m_0^s)s$$

  $\Rightarrow$ the matrix element of $-2O^{\Delta S=2}_{VA}$ in this theory gives the physical matrix element of $O^{\Delta S=2}_{VV}+O^{\Delta S=2}_{AA}$.

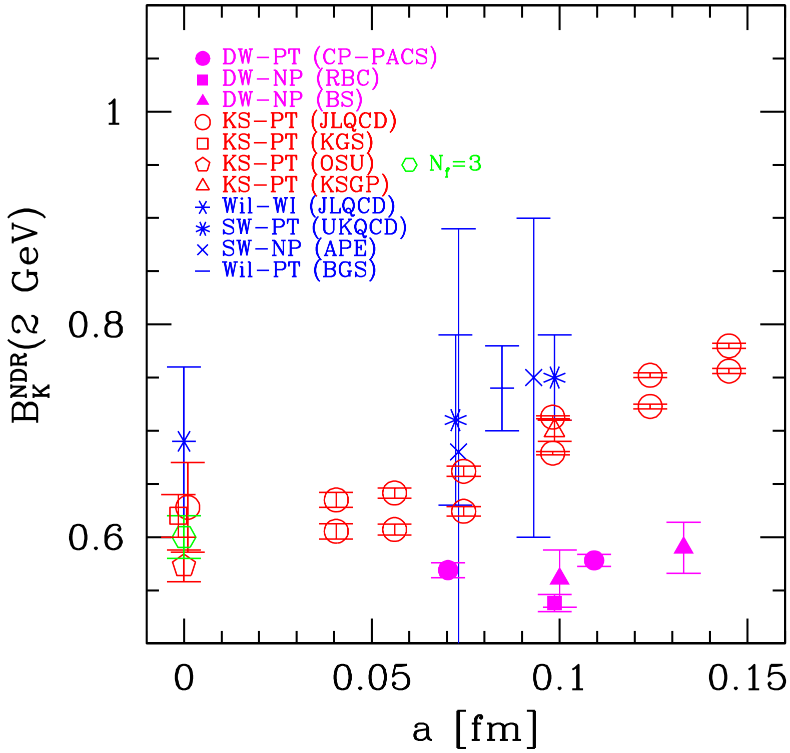
  R.Frezzotti, P.Grassi, S.Sint and P.Weisz (2000)

  2) It is also possible to use a (chiral) Ward Identity to determine the matrix element of $O^{\Delta S=2}_{VV}+O^{\Delta S=2}_{AA}$ from a measurement of that of $O^{\Delta S=2}_{VA}$ without the twisted mass-term.

  D.Becirevic et al. (2000)

$$B_K(2\,\text{GeV}) = 0.73 \pm 0.07^{+0.05}_{-0.01}.$$

- This calculation is also an excellent testing ground for simulations based on Domain Wall Fermions, Neuberger Fermions or other formulations of chiral fermions.

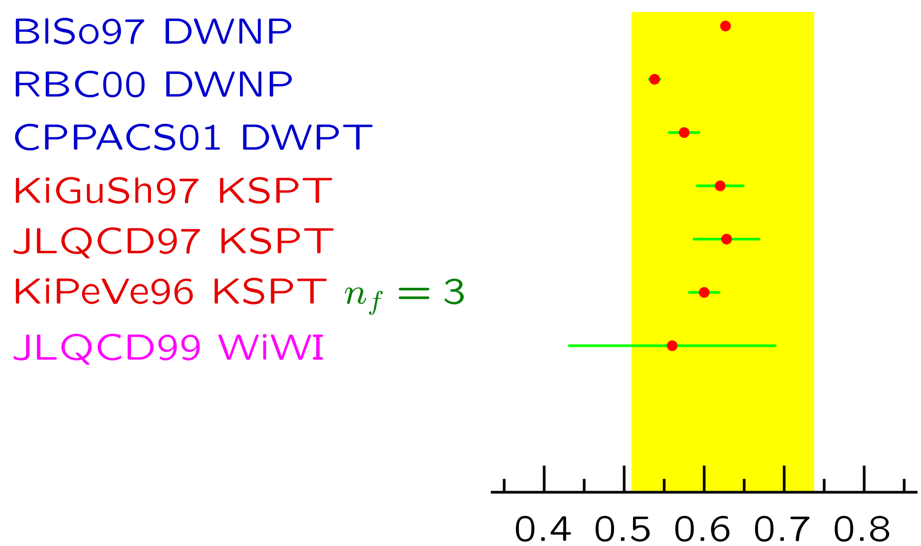


B_K^{NDR}(2 GeV) with various lattice actions as a function of the lattice spacing. All (bar one) of the simulations are quenched. From L.Lellouch - Latt00

D.Becirevic et al. (2000), do a quenched computation finding:

$$B_K(2\,\text{GeV}) = 0.73 \pm 0.07^{+0.05}_{-0.01}.$$

They also do an exploratory study with $N_f = 2$, and "do not see any significant deviation ...[from] the quenched simulation".



Results for $B_K^{NDR}(2\,GeV)$ in the continuum limit from various groups (mainly) in the quenched approximation. Statistical and systematic errors have been combined in quadrature (where these have been presented).

L. Lellouch, ("Light Hadron Weak Matrix Elements" Rapporteur at Lattice 2000):

- $$\begin{aligned} B_K^{\mathrm{NDR}}(2\,\mathrm{GeV}) &= 0.628 \pm 0.042 \pm 0.099 \\ \rightarrow \hat{B}_K^{\mathrm{NLO}} &= 0.86 \pm 0.06 \pm 0.14 \end{aligned}$$

- Matrix elements of $\Delta S = 2$ operators which appear in BSM are also being calculated. Donini et al. 1999

5. $K \to \pi\pi$ Decays

- At present we are unable to perform lattice calculations of exclusive two-body hadronic B-decays ($B \to \pi\pi, B \to K\pi$ etc.). Considerable progress is being made however, towards reliable calculations of $K \to \pi\pi$ decays, including quantitative studies of the $\Delta I = 1/2$ rule and an evaluation of ε'/ε.

- The ultra-violet problem, the construction of finite matrix elements of renormalized operators from the bare lattice ones, is in principle fully solved.
  M. Bocchicchio et al. (1985); L. Maiani et al. (1988); C. Bernard et al. (1988); C. Dawson et al. (1998).

- The infrared problem arises from two sources:
  i) The unavoidable continuation of the theory to Euclidean space-time. Maiani-Testa Theorem (1990).
  ii) The use of a finite volume in numerical simulations.

- An important step towards the solution of the IR problem has been achieved by L.Lellouch and M.Lüscher (2000), who derived a relation between the $K \to \pi\pi$ matrix elements in a finite volume and the physical amplitudes.

The LL-Relation

$$|\langle\, \pi\pi \,|\, \mathcal{H}_W(0) \,|\, K \,\rangle|^2 \;=\; 8\pi V^2 \{q\phi'(q) + k\delta_0'(k)\}_{k=k_\pi} \left(\frac{m_K}{k_\pi}\right)^3 |_V\langle\, \pi\pi \,|\, \mathcal{H}_W(0) \,|\, K \,\rangle_V|^2$$

- k is related to the centre of mass energy W by:

  $$W = 2\sqrt{m_\pi^2 + k^2} \quad \text{and} \quad k_\pi = \frac{1}{2}\sqrt{m_K^2 - 4m_\pi^2}\ .$$

  $q = kL/2\pi$ where L is the length of the lattice.

- $\phi(q)$ is a known kinematical function of q, and is a consequence of the shape of the (finite) lattice.

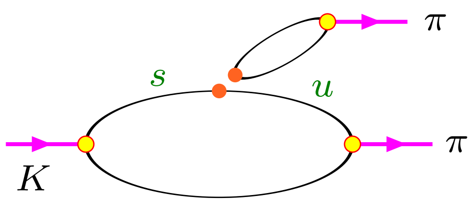
- $\delta_0(k)$ is the physical (infinite-volume) s-wave phase-shift (the explicit formula above assumes that only this phase-shift is non-zero).

- LL derive the formula for fixed large volume. We have rederived and reinterpreted it taking $V \to \infty$ at fixed "physics" starting from the quantization condition $n\pi = \phi(q) + \delta_0(k)$ (Lüscher 1986, 1991). The LL-factor has a very simple interpretation in terms of a "density of states", dn/dE, encountered when one related the sum over intermediate states in finite volume to the energy integral in infinite volume.

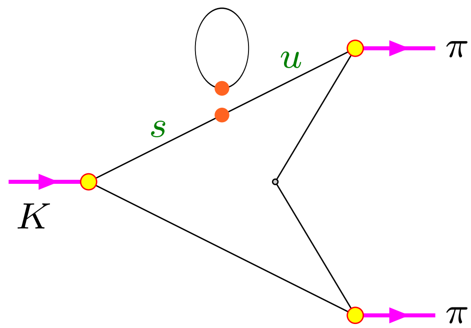
  Lin, Martinelli, CTS, Testa 2001

Prospects for Lattice Calculations $K \to \pi\pi$ Decays are exciting!

- A number of different mechanisms contribute to $K \to \pi\pi$ decay amplitudes, e.g.



Disconnected Emission



Disconnected Penguin

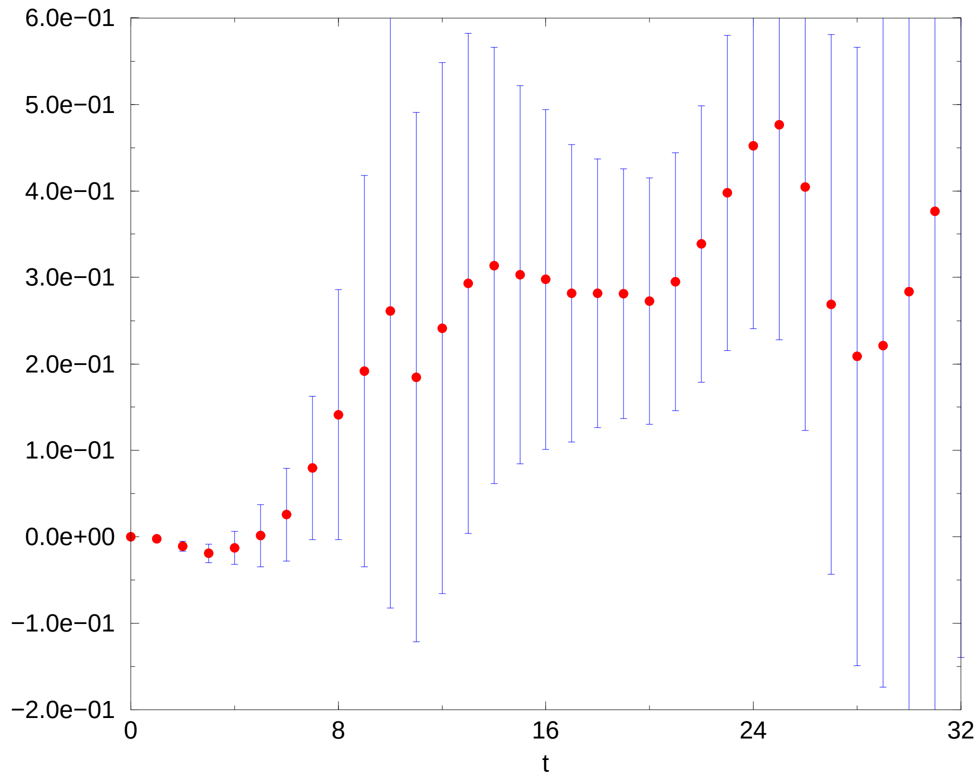
- Lattice calculations have shown that it is not possible to explain the $\Delta I{=}1/2$ rule with emission diagrams only.

- In order to obtain the physical contribution from the penguin diagrams in general we have to subtract large unphysical terms (power divergences). This is the reason for the absence up to now, of sufficiently precise results for $\Delta I = 1/2$ decays.

- There is a signal for $\Delta I = 1/2$ matrix elements
  $\Rightarrow$ data which we can analyse
  $\Rightarrow$ studies of $\Delta I = 1/2$ rule and evaluation of ε'/ε.

fig

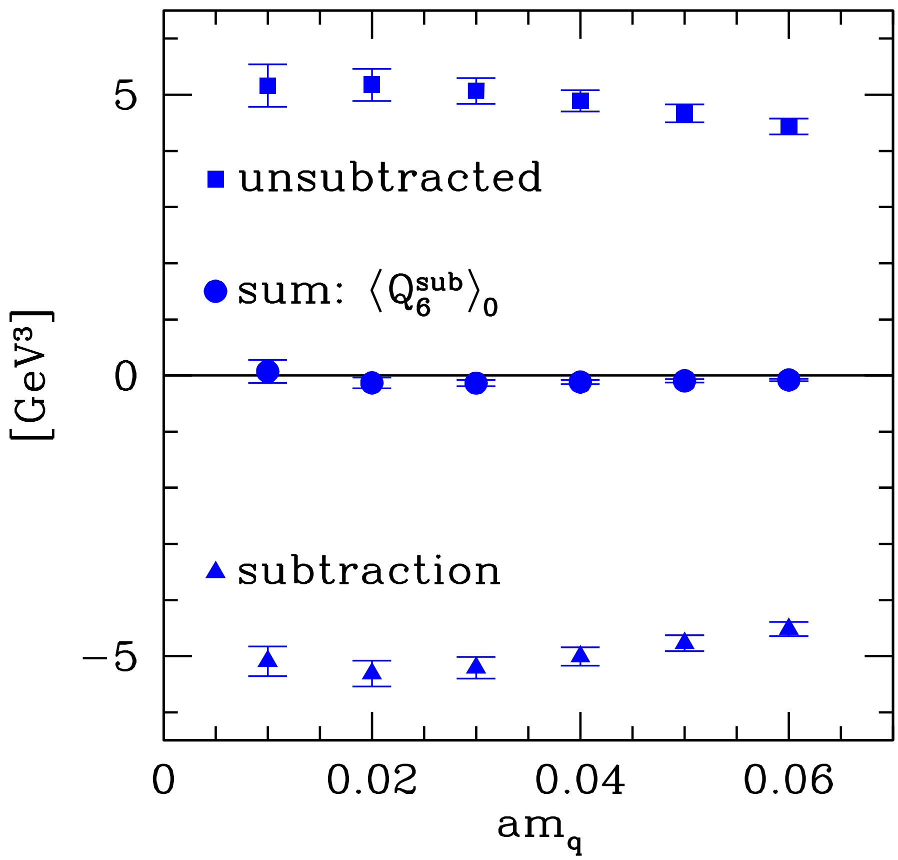
$K \to \pi\pi$ Decays - Cont:



Raw data for the matrix element ${}_{I=0}\langle\,\pi\pi\,|\,O^-\,|K\,\rangle$ as a function of the "time" of source of one of the pions.

PRELIMINARY! SPQ$_{\rm CD}$R Collaboration

$K \to \pi\pi$ **Decays - Cont:**

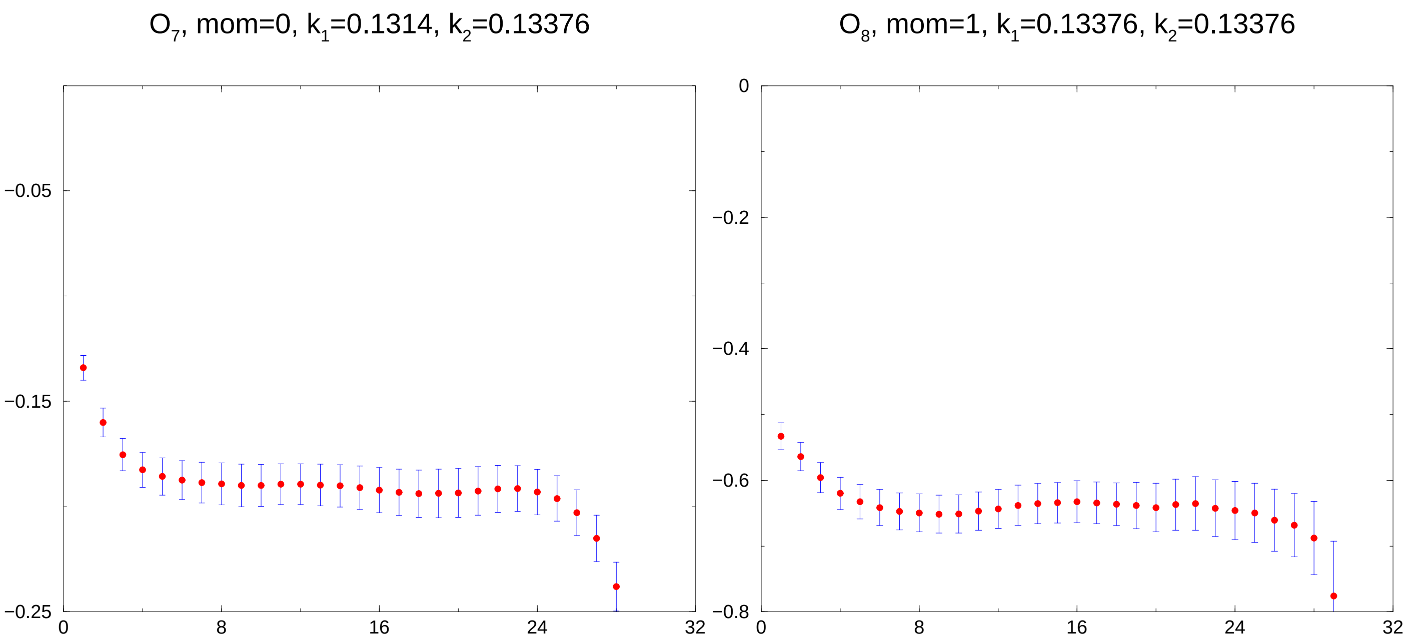


The values of the unsubtracted and subtracted bare matrix elements $_{I=0}\langle \pi \,|\, O_6 \,|K \rangle$ as a function of the quark mass.

CP-PACS:Lattice2000 Collaboration

$K \to \pi\pi$ Decays - Cont:

- Calculations of amplitudes for $\Delta I = 3/2$ transitions are relatively straightforward compared to $\Delta I = 1/2$ ones and the signals are very strong. We are currently undertaking a detailed study, up to NLO in the chiral expansion, for these matrix elements (Q_4, EW-penguins $Q_{7,8}$).



ΔI=3/2 matrix elements of the electroweak penguin operators O_7 (left) and O_8 (right).

PRELIMINARY! SPQ$_{\mathrm{CD}}$R Collaboration

$K \to \pi\pi$ Decays - Cont:

- The matrix element of Q_8 is important in the evaluation of ε'/ε. Lattice results (from $K \to \pi$ matrix elements + soft-pion theorems and χPT at leading order) are significantly smaller than other determinations, e.g. a lattice simulation gives

  $$|_{I=2}\langle\, \pi\pi \,|\, Q_8 \,|\, K^0 \,\rangle| = (0.5 \pm 0.01)\,\text{GeV}^3$$

  in the NDR renormalization scheme at $\mu = 2\,\text{GeV}$.

  A.Donini, V.Giménez, L.Giusti and G.Martinelli 1999

  This can be compared to:
  $1.3 \pm 0.3\,\text{GeV}^3$ (Donoghue and Golowich) and
  $3.5 \pm 1.1\,\text{GeV}^3$ (Knecht, Peris and de Rafael).
  Large matrix element of $O_8 \Rightarrow$ very large matrix element of Q_6 in order to explain the measured value of ε'/ε.

- Interesting studies using Domain Wall Fermions also underway (RBC, CP-PACS). $K \to \pi\pi$ amplitudes can be determined from $K \to \pi$ matrix elements if the chiral symmetry is sufficiently precise.

- Preliminary results indicate that it is possible to obtain a $\Delta I = 3/2$ $K \to \pi\pi$ decay amplitude which is consistent with experiment and yet with a large B_K (as indicated above).

Much exciting work do be done during the coming year or two!!!

6. Conclusions

- Lattice simulation provide the opportunity to evaluate non-perturbative QCD effects from first principles, with no model assumptions or parameters.

- There is a large range of quantities of central importance to particle physics which are being computed in lattice simulations (too large a range to be considered in a 1/2-hour talk).

- For some quantities (such as those which enter into the analysis of the Unitarity Triangle) the current emphasis is on the reduction of systematic errors. For others (e.g. nonleptonic weak decays) we are still learning how to extract the physical quantities. The range of quantities which can be studied is expanding.