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KIRCHHOFF'S APPROXIMATION FOR EVALUATING THE COUPLING OF DAΦNE RF CAVITY WITH WAVEGUIDE DAMPERS

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ABSTRACT

Waveguide dampers are widely used in accelerator RF cavities to reduce the quality factors of high order resonant modes, in order to avoid multibunch instabilities. The theoretical evaluation of the coupling between cavity and waveguide through large apertures presents numerous difficulties and requires a vast amount of computing time on powerful computers. In this paper we present a simple method to obtain a first estimate of the damped quality factors, based on the well-known Kirchhoff's approximation. We also compare our results with the measurements of the DAΦNE main rings cavity prototype and with the output data of two available computer codes.

1. INTRODUCTION

High order modes (HOMs) damping constitutes one of the main problems in the RF system of high current accelerators [1]. The beam spectrum may couple with the modes of the accelerating cavity, and if the quality factor Q of a given mode is high enough (i.e. if the parasitic losses are low with respect to the energy left by each passing bunch) the beam can become unstable.

Two methods are currently used to extract the HOM power from the RF cavity in order to attenuate such instability: antenna or loop couplers and waveguide couplers. Both reduce the shunt impedance of the cavity HOMs; the waveguides have an advantage over loop couplers in that they offer an intrinsic rejection of the accelerating mode of the cavity, while a loop coupler needs some kind of filtering. Moreover, waveguide dampers work on a much wider frequency range. On the contrary an antenna or magnetic loop damper applied on the resonator surface in correspondence of the peak of a specific HOM, offers a better damping for that mode.

In the case of DAΦNE [2], where broadband damping is required, waveguides have been chosen; the residual beam oscillations are damped by a bunch-by-bunch digital feedback system. With this feedback system alone, because of the high current stored in the machine (1.5÷5 Ampere), it would be impossible to stabilize the beam.

In this paper we investigate the problem of the calculation of the coupling between an accelerating cavity and waveguide dampers. We restricted ourselves to an approximate method, comparing the results to the measurements done on the DAΦNE RF cavity prototype; further comparison is made with HFSS [3] and POPBCI [4] simulation codes.

2. KIRCHHOFF'S APPROXIMATION

2.1 High order modes damping

In studying multibunch instabilities for DAΦNE, we have found that, even taking into account the damping mechanisms (e.g. feedback systems, radiation and Landau damping), the beam is unstable unless the Q values of the HOMs are kept below a certain threshold. The design of waveguide damping systems requires to evaluate the coupling between cavity and waveguides. This can be done by using three-dimensional computer simulation codes, which are very time consuming even on powerful computers; on the other side direct measurements on prototypes are not very practical as well. Therefore a simple way of evaluating Qs, even if approximate to some extent, would be quite useful.

Monopolar modes have the highest values of coupling impedance since they are the only modes with a non-zero longitudinal electric field on the cavity axis. Being azimuthally symmetric, it is possible to calculate them with a simple 2D simulation code. The output of such a code may then be used to evaluate the additional losses due to radiation in the waveguides.

The electromagnetic field propagating into a waveguide connected to the resonant cavity through an aperture A can be expressed in terms of the waveguide Green's functions and of the tangential field on A:

$$\begin{aligned}\mathbf{E}_{wg}(\mathbf{r}) &= -j\omega\mu\int_A \overline{\mathbf{G}}_1(\mathbf{r},\mathbf{r}')\cdot\mathbf{z}_0(\mathbf{r}')\times\mathbf{H}(\mathbf{r}')\,dA' + \int_A \overline{\mathbf{G}}_3(\mathbf{r},\mathbf{r}')\cdot\mathbf{z}_0(\mathbf{r}')\times\mathbf{E}(\mathbf{r}')\,dA' \\ \mathbf{H}_{wg}(\mathbf{r}) &= j\omega\varepsilon\int_A \overline{\mathbf{G}}_2(\mathbf{r},\mathbf{r}')\cdot\mathbf{z}_0(\mathbf{r}')\times\mathbf{E}(\mathbf{r}')\,dA' + \int_A \overline{\mathbf{G}}_4(\mathbf{r},\mathbf{r}')\cdot\mathbf{z}_0(\mathbf{r}')\times\mathbf{H}(\mathbf{r}')\,dA'\end{aligned} \tag{1}$$

where $\mathbf{z}_0$ is the unit vector normal to A and, in our case, has the same direction as the waveguide axis; the dyadics $\overline{\mathbf{G}}_i$ for the rectangular waveguide, are known from literature [5].

If the slot dimensions are comparable to the wavelength of the HOM considered, it is reasonable to use equations (1) to calculate the field propagating in the waveguide. We can replace the exact fields in the integrals with the uncoupled cavity field values ($\mathbf{E}_{c0}$,$\mathbf{H}_{c0}$) on A: this procedure is known as Kirchhoff's approximation.

As the electric field $\mathbf{E}_{c0}$ is obviously normal to A, we can rewrite equations (1) as:

$$\begin{aligned}\mathbf{E}_{wg}(\mathbf{r}) &= -j\omega\mu\int_A \overline{\mathbf{G}}_1(\mathbf{r},\mathbf{r}')\cdot\mathbf{z}_0(\mathbf{r}')\times\mathbf{H}_{c0}(\mathbf{r}')\ dA' \\ \mathbf{H}_{wg}(\mathbf{r}) &= \int_A \overline{\mathbf{G}}_4(\mathbf{r},\mathbf{r}')\cdot\mathbf{z}_0(\mathbf{r}')\times\mathbf{H}_{c0}(\mathbf{r}')\ dA'\end{aligned} \tag{2}$$

The power absorbed by the waveguide is given by the real part of the Poynting vector flux through the aperture:

$$P_{wg} = \frac{1}{2}\mathrm{Re}\left\{\int_A \mathbf{E}_{wg}\times\mathbf{H}^*_{wg}\cdot\mathbf{z}_0\ dA\right\} \tag{3}$$

The Q of the loaded cavity (Q_L) is therefore:

$$Q_0 = \frac{\omega U}{P_0} \Rightarrow Q_L = \frac{\omega U}{P_0 + P_{wg}} = \frac{1}{Q_0^{-1} + \dfrac{P_{wg}}{\omega U}} \tag{4}$$

where Q_0 is the unloaded quality factor, P_0 is the power dissipated by the cavity walls, ω is the angular resonant frequency and U is the energy stored in the cavity.

The quantity $\omega U/P_{wg}$ is often referred to as Q_{ext}. If the losses on the walls are considerably less then the losses caused by the waveguides (i.e. if the Q_0 is high enough), then $Q_L \approx Q_{ext}$.

We have applied this method to the case of the DAΦNE main rings cavities, where a total of five rectangular waveguides have been used. Three 305×40 mm^2 guides are placed 120° apart onto the cavity main body and two smaller 140×40 mm^2 guides are applied on the tapers, rotated 90° apart for a better dipole coupling.

The cut-off frequency of the bigger waveguides is 491 MHz, while the next two propagating modes are at 982 MHz (TE_{20}) and 1.474 GHz (TE_{30}) and the cut-off of the small waveguides is at 1.070 GHz.

The accelerating mode of the cavity (0-EM-1) resonates at 367 MHz and does not propagate in the waveguides; we shall see later how it is affected by their presence.

Thus, in expressions (2) we can restrict ourselves to a modal expansion of the Green's functions consisting of the TE_{10} mode alone when dealing with HOMs below the TE_{20} cut-off, since all the other modes give no contribution to (3).

The rectangular waveguide Green's functions to use in (2) are in conclusion:

$$\overline{\mathbf{G}}_1(\mathbf{r},\mathbf{r}') = -\frac{1}{ab}\frac{e^{jk_z(z-z')}}{jk_z}\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{a}x'\right)\mathbf{y}_0\mathbf{y}_0$$

$$\overline{\mathbf{G}}_4(\mathbf{r},\mathbf{r}') = -\frac{1}{ab}\frac{e^{jk_z(z-z')}}{jk_z}\left[-jk_z\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{a}x'\right)\mathbf{x}_0\mathbf{y}_0 + \right. \qquad (5)$$

$$\left. +\frac{\pi}{a}\cos\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{a}x'\right)\mathbf{z}_0\mathbf{y}_0\right]$$

where $\mathbf{x}_0\mathbf{y}_0$, $\mathbf{y}_0\mathbf{y}_0$ and $\mathbf{z}_0\mathbf{y}_0$ are the ordinary Cartesian tensor components.

Substituting (2) and (5) in expression (3), we have:

$$P_{wg} = \frac{1}{2}\frac{\omega\mu}{k_z}\frac{1}{(ab)^2}\int_0^a\int_0^b \sin^2\left(\frac{\pi}{a}x\right)dx\,dy\,\cdot \qquad (6)$$

$$\cdot\left[\int_0^a\int_0^b \sin\left(\frac{\pi}{a}x'\right)H_{0x}(x',y')\,dx'dy'\right]^2$$

where $\mathbf{H}_{c0}=H_{0x}\mathbf{x}_0+H_{0y}\mathbf{y}_0$. In the case of N rectangular waveguides $a\times b$, expression (4) becomes:

$$Q_L = \left\{Q_0^{-1} + \frac{\frac{N}{4}\frac{\omega\mu}{k_z}\frac{1}{ab}\left[\int_0^a\int_0^b \sin\left(\frac{\pi}{a}x'\right)H_{0x}(x',y')\,dx'dy'\right]^2}{\omega U}\right\}^{-1} \qquad (7)$$

We note that only the component H_{0x} of the cavity magnetic field couples with the waveguide TE_{10} field (this is true in general for all the TE_{m0} modes). In addition it is easy to show that the coupling to the TE_{20} is negligible, since the coupled power depends on the integral:

$$\int_0^a\int_0^b \sin\left(\frac{2\pi}{a}x'\right)H_{0x}(x',y')\,dx'dy' \qquad (8)$$

which is zero given the azimuthal symmetry of H_{0x} and the orientation of the slots (Fig. 1). We shall therefore consider the coupling of all cavity modes up to 1.5 GHz with the TE_{10} only. Above this frequency the wall losses and the effect of the propagation through the beam tubes decrease the HOM Qs significantly.

In the more general case of coupling to several waveguide modes, this procedure keeps valid since the modes transport power independently. The total power will be given by the sum of the (6) where the proper expressions of the Green's function component have to be used.

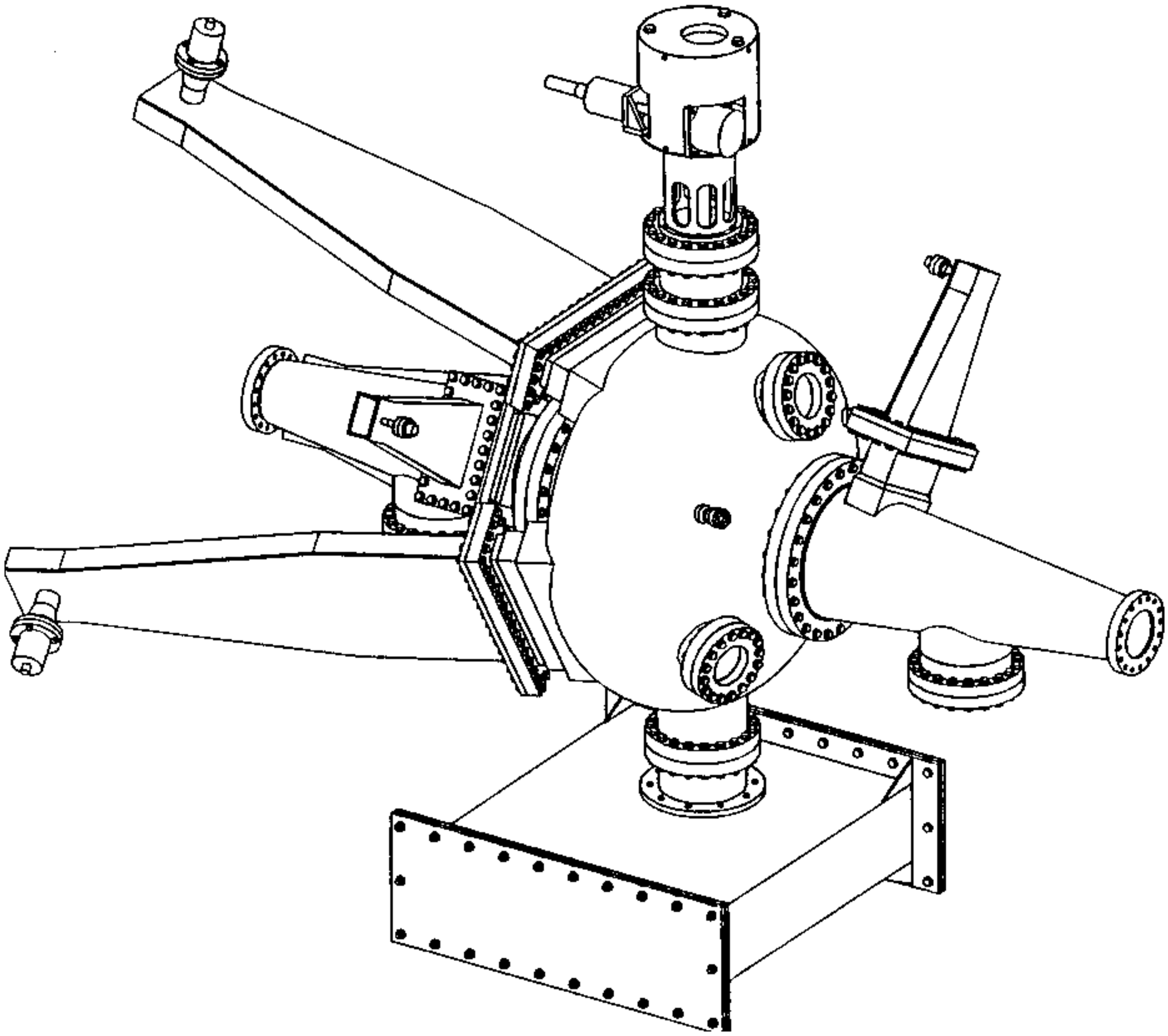

Fig. 1 - DAΦNE main ring cavity.

2.2 Losses for the accelerating mode

The measurements performed on the cavity prototype have shown a small damping effect on the accelerating mode due to after the application of the waveguide dampers.

Since at the 0-EM-1 frequency no mode propagates into the waveguide, the attenuation is due to the power dissipation of the evanescent modes on the waveguides walls. The waveguide modes below cut-off are attenuated exponentially with a constant

$$\alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2} = -jk_z \qquad (9)$$

and they enter the waveguides for a few wavelengths.

Also in this case the main contribution to these power losses is given by the TE_{10}, which has the lowest attenuation constant. It is worth noting that expression (9), valid only in the loss-free case, can be used since the attenuation due to wall losses is much lower than the attenuation given by the cut-off condition.

The power dissipated on the waveguide walls is given by the real part of the Poynting vector flux normal to the metallic surfaces:

$$P_d = \frac{1}{2}\,\mathrm{Re}\left\{\oint_C \int_0^\infty \mathbf{E}_\tau \times \mathbf{H}_\tau \cdot \mathbf{n}_0 \; dz\,ds\right\} \tag{10}$$

where C is the waveguide section boundary.

Since on the surface of a good conductor (having conductivity equal to σ) the Leontovic's relation holds:

$$\mathbf{E}_\tau = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}}\,\mathbf{H}_\tau \times \mathbf{n}_0 \tag{11}$$

equation (10) becomes:

$$P_d = \sqrt{\frac{\omega\mu}{8\sigma}} \oint_C \int_0^\infty \left|\mathbf{H}_\tau\right|^2 dz\,ds \tag{12}$$

From the second of (5) we can obtain the components of the magnetic field in the waveguides:

$$\begin{aligned} H_x &= -\frac{1}{ab} e^{jk_z z} \sin\left(\frac{\pi}{a}x\right) \int_0^a \int_0^b \sin\left(\frac{\pi}{a}x'\right) H_{0x}(x',y')\; dx'dy' \\ H_z &= -\frac{\pi}{a^2 b}\frac{e^{jk_z z}}{jk_z} \cos\left(\frac{\pi}{a}x\right) \int_0^a \int_0^b \sin\left(\frac{\pi}{a}x'\right) H_{0x}(x',y')\; dx'dy' \end{aligned} \tag{13}$$

We note that H_X vanishes on the walls (x=0 and x=a) while it is normal to the other two waveguide walls. Therefore only the component H_z is considered in calculating the power loss.

In conclusion, we find for a single waveguide:

$$P_d = \sqrt{\frac{\omega\mu}{8\sigma}}\,\frac{\pi^2}{a^3 b}\left(\frac{1}{a}+\frac{1}{2b}\right)\frac{\left[\int_0^a \int_0^b \sin\left(\frac{\pi}{a}x'\right) H_{0x}(x',y')\; dx'dy'\right]^2}{\alpha^3} \tag{14}$$

3. ANALYTICAL AND EXPERIMENTAL RESULTS

To calculate the integrals in formulas (7) and (14), the magnetic field H_{0x} has been approximated by a third order polynomial fitting the output data of the OSCAR2D [6] two-dimensional simulation code for each mode.

The experimental measurements have been performed on a low cost copper prototype of the RF cavity. The resonance frequencies and the Q_0 values reported in Tab. 1 show significant differences between the numerical simulation (URMEL code [7]) and the measurements [8] because of the poor quality of the prototype.

Table 1 - Unloaded cavity

MODE	Freq. [MHz] (URMEL)	Freq. [MHz] (Measured)	Q_0 (URMEL)	Q_0 (Measured)
0-EM-1	367.4	357.0	49100	25000
0-MM-1	696.0	747.5	49800	24000
0-EM-2	794.9	796.8	81900	40000
0-MM-2	987.2	1023.6	65900	28000
0-EM-3	1069.8	1121.1	66900	12000
0-MM-3	1138.4	1175.9	56800	5000
0-EM-4	1119.9	1201.5	57500	9000
0-EM-5	1203.8	1369.0	67600	5000

However, since the Q_0 values are high enough, according to (7), these differences are not important when we compare measured and calculated Q_L. In the accelerating mode case it is instead necessary to take into account the differences on the Q_0 values, and only a comparison between the relative reduction (i.e. the Q_0/Q_L ratio) is significative.

In Tab. 2 we show the results obtained from both measurements and Kirchhoff's approximation.

Table 2 - Waveguides coupled cavity (*) Q_0/Q_L

MODE	Freq. [MHz] (Measured)	Q_L (Calc.)	Q_L (Measured)
0-EM-1	349.5	1.07*	1.14*
0-MM-1	745.7	75	70
0-EM-2	796.5	550	230
0-MM-2	1024.9	190	150
0-EM-3	1125.4	—	240
0-MM-3	1172.0	65	100
0-EM-4	1194.3	220	130
0-EM-5	1361.6	115	300

We see that there is a rather good agreement between theoretical previsions and experimental measurements. However, we expect this accuracy to reduce for the higher frequency HOMs, as the fringing effects become more evident.

The 0-EM-3 mode deserves particular attention since from Tab. 1 one can see that the calculated resonance frequency, used in Kirchhoff's approximation, is very close to the cut-off frequency of the smaller waveguides (the difference is in fact less than 0.1 %). In this case equation (7) shows a singularity since $k_z \rightarrow 0$, yielding a gross overestimate of the damping effect.

Tab. 3 shows a comparison of Kirchhoff's method with results of the two available simulation codes HFSS and POPBCI [9]. Since those codes run a cavity shape slightly different from the prototype (the waveguides are placed on the maximum peak of the 0-MM-1 mode), calculations have been performed also for the new geometry, obtaining values of Q_L different from those of Tab. 2.

Table 3 - Computer simulations

MODE	$\mathbf{Q_L}$ (Calc.)	$\mathbf{Q_L}$ (HFSS)	$\mathbf{Q_L}$ (POPBCI)
0-EM-1	1.08*	1.29*	1.34*
0-MM-1	48	28	10
0-EM-2	2071	1136	—
0-MM-2	1030	73	—

It must be noted that in the case of HOMs having almost vanishing magnetic field on the surface A (as is the case of the MM-2 mode in Tab. 3) the perturbation induced by the waveguides on the magnetic field is no longer negligible, so that Kirchhoff's approximation, based on the unperturbed fields, might fail.

4. CONCLUSIONS

In this note we derive a semianalytical method to estimate the coupling of a resonant cavity to absorbing waveguides, based on Kirchhoff's approximation. The method can be easily used to evaluate the quality factor of damped HOMs, when the RF cavity is loaded with waveguides through large apertures. The comparison with measurements and computer simulations shows a rather good agreement, and makes this approximate method useful, at least at a first stage of the waveguide couplers design.

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