DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

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Note: **L-37**

AN OPTICAL MODEL FOR OFF-AXIS QUADRUPOLES

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1. Description of the method

The description in terms of standard optical elements of the Interaction Region (IR) quadrupoles where the electron and positron beams travel off-axis in a common vacuum chamber has been a problem since the first design of DAΦNE. In this note a new approach is described, which takes into account the magnetic measurements performed on the quadrupole. Particles are tracked through the measured field along the nominal trajectory and with small displacements and angles to obtain the transfer matrix of the quadrupole. Then the resulting matrix is fitted by a proper combination of standard optical elements such as drift spaces and bending dipoles with gradient. The results of the procedure performed on one of the permanent magnet quadrupoles of the KLOE interaction region are presented.

2. Application to the lattice proposed for the DAΦNE upgrade

The IR lattice for an upgrade of DAΦNE by means of large crossing angle, small betatron functions at the Interaction Point (IP) and the "crabbed waist" scheme [1] consists of a short vacuum chamber common to both beams surrounded by a first vertically focusing permanent magnet quadrupole on each side of the IP, followed by separated channels with 4 small horizontally focusing quads. The total crossing angle is 50 mrad (25 mrad per beam) and the first quadrupole is 25 cm long, starting at 0.3 m from the IP, and its expected deflection is 50 mrad.

In order to test the procedure the measured field of the third quadrupole of the first KLOE IR has been used, since its magnetic length is ≈27 cm, similar to the one proposed for the new structure. However, its bore diameter is much larger and therefore the shape of the gradient in the new lattice should exhibit a more pronounced flat top. Fig.1 and 2 show the measured gradient of the KLOE QF2 permanent magnet quadrupole and its third order component. The measured field map on the horizontal symmetry plane spans from -30 mm to +30mm from the axis in steps of 10 mm, and from -450 mm to 450 mm in the longitudinal direction. The field quality is very good, the third order component being negligible for the purpose of this example. Particle tracking has been performed following the method described in [2], where the quadratic term has been neglected.

The quadrupole field has been positioned with the maximum gradient at 425 mm from the IP with the axis along the IR symmetry line. The nominal particle starting from the IP with an angle of 25 mrad with respect to the quadrupole axis has been tracked through the field, multiplying all field values by the same factor until the final angle of the trajectory at 850 mm from the IP has reached the nominal value of 75 mrad.

The initial positions of the particles lie on a straight line at the IP perpendicular to the nominal trajectory at 25 mrad and tracking is stopped when the particles reach another straight line passing through the final position of the nominal trajectory at 75 mrad with respect to the quadrupole axis and perpendicular to it.

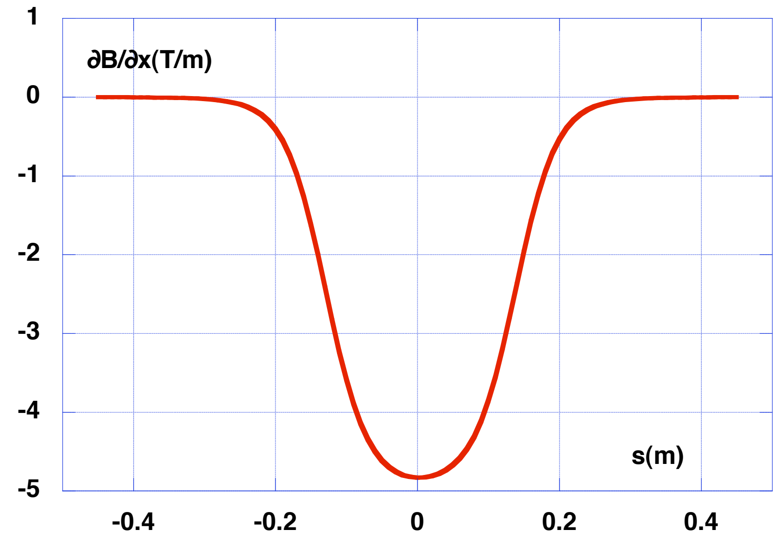

Figure 1 - First order component of the horizontal field expansion of the KLOE QF2 permanent magnet quadrupole.

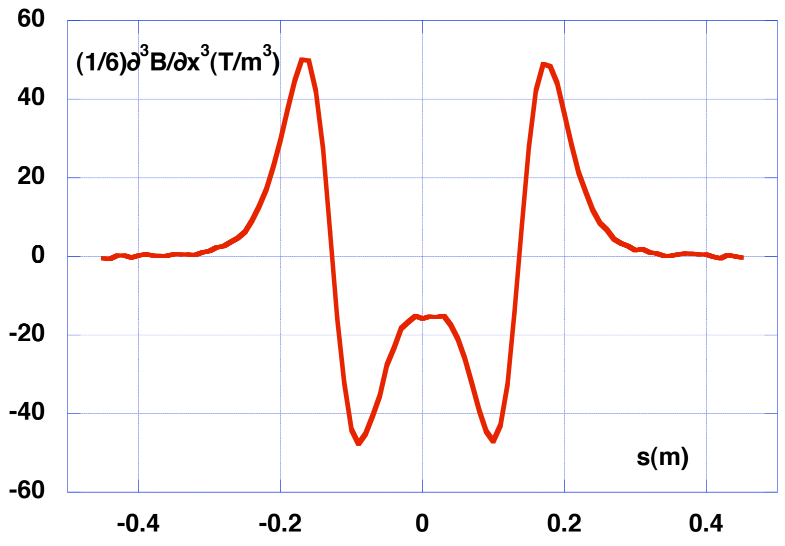

Figure 2 - Third order component of the horizontal field expansion of the KLOE QF2 permanent magnet quadrupole.

The transfer matrix of the quadrupole around the nominal trajectory has been obtained by propagating particles with ±1 mm displacement, ±1 mrad angle and ±0.1% energy deviation with respect to the nominal trajectory. The result is the following 5x5 matrix:

$$\begin{vmatrix} 2.96059 & 1.63255 & -0.02222 & 0 & 0 \\ 4.84198 & 3.00806 & -0.05826 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.39669 & 0.26203 \\ 0 & 0 & 0 & -3.17862 & -0.42836 \end{vmatrix}$$

For comparison the corresponding matrix around the quadrupole axis comes out to be:

$$\begin{vmatrix} 2.95345 & 1.62690 & 0 & 0 & 0 \\ 4.82380 & 2.99575 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.39236 & 0.26402 \\ 0 & 0 & 0 & -3.16324 & -0.42011 \end{vmatrix}$$

The integrated gradient on axis corresponding to the desired 50 mrad deflection is 6.686 T, corresponding to K_{MAD}=15.73 m^{-2} over the design quadrupole length of 250 mm. The nominal trajectory is 1.18 mm longer than its projection on the quadrupole axis. The determinant of the horizontal part of the first matrix is 1.00086, 1.00282 for the vertical, while those of the matrix on axis are 0.99996 and 0.99999 respectively.

3. Optical model

Being the diagonal elements of the matrix slightly different from each other, it has been decided to fit the matrix by means of a structure consisting of a dipole with gradient surrounded by two straight sections of different length. By setting:

$$\gamma = \sqrt{n-1}\alpha$$

$$\alpha = \frac{L_B}{\rho}$$

where α is the bending angle of the dipole, L_B its magnetic length, ρ its bending radius and n its field index, related to the MAD constant by:

$$n = -K_{MAD}\rho^2$$

the horizontal elements of the structure, neglecting the edge focusing of the dipole, are:

$$a_{11} = \cosh(\gamma) + \frac{L_2\sqrt{n-1}}{\rho}\sinh(\gamma)$$
$$a_{12} = (L_1 + L_2)\cosh(\gamma) + \left(\frac{L_1 L_2\sqrt{n-1}}{\rho} + \frac{\rho}{\sqrt{n-1}}\right)\sinh(\gamma)$$
$$a_{13} = -\frac{\rho}{n-1}\left[1-\cosh(\gamma)\right] + \frac{L_2}{\sqrt{n-1}}\sinh(\gamma)$$
$$a_{21} = \frac{\sqrt{n-1}}{\rho}\sinh(\gamma)$$
$$a_{22} = \cosh(\gamma) + \frac{L_1\sqrt{n-1}}{\rho}\sinh(\gamma)$$
$$a_{23} = \frac{1}{\sqrt{n-1}}\sinh(\gamma)$$

while the vertical ones, with $\gamma = \sqrt{n}\alpha$, are:

$$a_{44} = \cos(\gamma) - \frac{L_2\sqrt{n}}{\rho}\sin(\gamma)$$
$$a_{45} = (L_1 + L_2)\cos(\gamma) + \left(\frac{\rho}{\sqrt{n}} - \frac{L_1 L_2\sqrt{n}}{\rho}\right)\sin(\gamma)$$
$$a_{54} = -\frac{\sqrt{n}}{\rho}\sin(\gamma)$$
$$a_{55} = \cos(\gamma) - \frac{L_1\sqrt{n}}{\rho}\sin(\gamma)$$

Being n of the order of several hundreds, it is possible to neglect 1 with respect to n and therefore find γ from the ratio a_{21}/a_{54}, The knowledge of n, ρ and L_B comes then from the condition of a bending angle of 50 mrad and the values of a_{21} and a_{54} . From the matrix values I find:

$$\gamma = 1.1218$$
$$n = 503.37$$
$$\rho = 6.3588m$$
$$L_{\mathbf{B}} = 0.3179m$$

It is possible now to calculate the values of L_1 and L_2 from a_{11} and a_{22}, which, in principle, could be different from those calculated from a_{44} and a_{55}. In fact the result in the first case is:

$$L_1 = 0.27055m$$
$$L_2 = 0.26075m$$

while in the second:

$$L_1 = 0.27132m$$
$$L_2 = 0.26136m$$

Taking the average values:

$$L_1 = 0.27093m$$
$$L_2 = 0.26105m$$

it is possible to calculate the full matrix:

$$\begin{vmatrix} 2.95713 & 1.63275 & -0.02477 & 0 & 0 \\ 4.82971 & 3.00484 & -0.06113 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.39554 & 0.26149 \\ 0 & 0 & 0 & -3.17839 & -0.42694 \end{vmatrix}$$

The difference from the original matrix, in percentage, is:

$$\begin{vmatrix} 0.12 & 0.01 & 11.48 & 0 & 0 \\ -0.25 & -0.11 & 4.93 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.29 & -0.21 \\ 0 & 0 & 0 & -0.01 & -0.33 \end{vmatrix}$$

For the betatron part the difference is of the same order of the error on the determinant of the matrix calculated from tracking, making therefore a further improvement of the fit by means of the focusing edges of the dipole useless. In the case of the dispersive terms the fractional difference is large, but the absolute value negligible in practice with respect to typical dispersion errors. In fact:

$$\Delta a_{13} = 2.6mm$$
$$\Delta a_{23} = 2.9mrad$$

With the values found for the optical model, the integrated gradient in the dipole is:

$$L_B \frac{\partial B}{\partial x} = \frac{(B\rho)L_B n}{\rho^2} = 6.728T$$

0.6% larger than that calculated on the quadrupole axis. The total path length in the model is 0.8499 m, 1.3 mm shorter than the one from tracking on the nominal trajectory. The horizontal position in the model at the end of the structure is 42.3 mm, while tracking gives 40.8 mm.

References

[1] Proposal for the DAΦNE upgrade. To be published.
[2] M. Preger, 'The wiggler transfer matrix', DAΦNE Technical Note L-34 (18/11/2003).