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Calculation of Touschek Effects for DAFNE with Strong RF Focusing*

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Introduction

Our aim is to calculate the steady-state emittance and energy spread as well as the beam life-time for DAFNE magnetic lattice modification designed for realization of the Strong RF Focusing Experiment concept (SRFF) proposed and developed in Frascati [1,2]. SRFF regime is based on a strong rf voltage and a high momentum compaction ring lattice which together produce a bunch length modulation along the ring. Also such a modulation can be organized in a ring structure where the dependence of the longitudinal position of a particle on its energy along the ring oscillates between large positive and negative values as it has been suggested recently in [3]. This gives a possibility simultaneously to obtain a smaller beam length and to decrease the synchrotron tune controlled by means of the rf voltage and by the momentum compaction. It is a way to overcome one of the critical points of the SRFF principle relating to a negative role of large synchrotron tunes in the viewpoint of both the dynamic aperture and beam-beam effects [2,3].

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Because of a low DAFNE energy, the influence of IBS (Intra-Beam Scattering, otherwise Touschek) effects [4] in our consideration is determinative. By this reason, the variations of the betatron coupling parameter as well as of the bunch current (a beam population number) are important to choose most reasonable conditions for a test SRFF experiment. In a whole, the task is of general interest due to the strong bunch length modulation and the energy aperture (connecting with the dynamic aperture) variation along a SRFF machine. So in order to obtain a correct result one must:

i) take into account conjointly the single and multiple IBS processes;

ii) consider variability of the beam length and of the energy aperture (EA) with the machine azimuth;

iii) make calculations for sufficiently wide range of changes in betatron coupling and the beam current per bunch.

We perform necessary calculations using the following computing tools:

- 6D Particle Tracking Code to calculate the dynamic aperture (DA) [5];

- the code to calculate IBS influence taking into account two-dimensional character of particle collisions inside a beam [6,7].

Below we describe the methods applied for these codes. Touschek effects were calculated for three versions of SRFF DAFNE magnetic lattice [2]. Calculation results are represented and discussed.

1 Azimuth-dependent Beam Length

According to the approach developed in [1] the beam length σ_Z in a SRFF machine is a function of the azimuth (s):

$$\sigma_Z(s) = \sigma_E \sqrt{\frac{|\alpha| L}{2\pi} \frac{E\lambda_{RF}}{eV_{RF}} \left[1 - 2\pi \frac{R_{56}(s)}{\lambda_{RF}} \cdot \frac{eV_{RF}}{E} (1 - \frac{R_{56}(s)}{\alpha L}) \right]}$$

Here, the $R_{56}(s) = \int_0^s h(s')\eta_X(s')ds'$ parameter relates the path length to the energy deviation of a particle, $\eta_X(s)$ is the horizontal dispersion function, $\alpha = R_{56}(L)/L$ is the momentum compaction factor, L is the machine perimeter, V_{RF} and λ_{RF} are respectively the rf voltage amplitude and wave length.

On the contrary, the relative energy spread σ_E does not vary along the ring but its value is modified due to Strong RF focusing. In the simplified assumption of constant bending radius and R_{56} linearly growing in the arcs the energy spread can be expressed as

$$\sigma_E = \sigma_E^{(0)} \sqrt{\frac{2}{3} \cdot \frac{(2 + \cos\mu)}{(1 + \cos\mu)}},$$

$$\cos\mu = 1 - \pi \frac{\alpha L}{\lambda_{RF}} \frac{eV_{RF}}{E}.$$

The quantity $\sigma_E^{(0)}$ is the relative energy spread in the "Weak" RF focusing limit. In the DAFNE Concept of RF Strong Focusing experiment the minimal beam length is achieved in IP (Interaction Point):

$$\sigma_Z^{min} = \sigma_Z(IP) = |\alpha| L \sigma_E^{(0)} \sqrt{\frac{2+\cos\mu}{6(1-\cos\mu)}}.$$

The maximal beam length is at the azimuth of Strong RF Focusing cavity position and its ratio to the minimal length is:

$$\frac{\sigma_Z^{max}}{\sigma_Z(IP)} = \frac{\sigma_Z(cav)}{\sigma_Z(IP)} = \frac{\sqrt{2(1-\cos\mu)}}{\sin\mu}.$$

This ratio increases at $\mu \to \pi$, i.e. with strengthening RF focusing. Note that such a modulation of the beam length allows sumultaneously to provide an opportunity to rise a machine luminosity (with decreasing the beta function at IP) and to avoid a significant increase of the particle loss rate in Touschek processes depending on the beam length averaged over the ring. (At that, to exclude the effects of microwave instabilities one must place the high impedance elements in the ring zones corresponding to the longer bunch. The same principle can be applied also to rings designed for the coherent SR production [2].)

2 Touschek effects in 2D collision approach

Below we briefly describe the method developed in [6,7] and tested in comparison with experimental data at VEPP-4M and CESR [7]. Based on theory given in [6] it expands one-dimensional (flat beam) approximation into theory which takes into account two-dimensional character of two-particle Coulomb interaction inside a bunch. With this aim the coupling parameter in velocity space $k = \sigma_{X'}/\sigma_{Y'}$ was introduced, where $\sigma_{X'}$ and $\sigma_{Y'}$ are respectively the spreads of trajectory angles in the horizontal plane (X) and the vertical one (Y). In the so-called "round" beam $k \to 1$ and in the flat one $k \to \infty$. The modified function of distribution as to momentum (p) in the center-of-mass system (CMS) has the following form [6,7]:

$$f(k,p)dp = \frac{2kp}{\sigma_p^2} \cdot S(w,k)dp, \quad (p>0)$$

$$S(w,k) = \exp\left[-\frac{w}{2}(1+k^2)\right] I_0 \left[\frac{w}{2}(1-k^2)\right]. \qquad (1)$$

Here $p = m\nu/2$, the momentum in CMS (similarily to [4], we use the non-relativistic description of motion in CMS); m is the rest electron mass; ν is the relative velocity of colliding particles inside a beam $(\nu^2 = \nu_X^2 + \nu_Y^2)$; $w = p^2/\sigma_p^2$; $\sigma_p = mc\gamma\sqrt{\sigma_{X'}^2 + \sigma_{Y'}^2}$, the

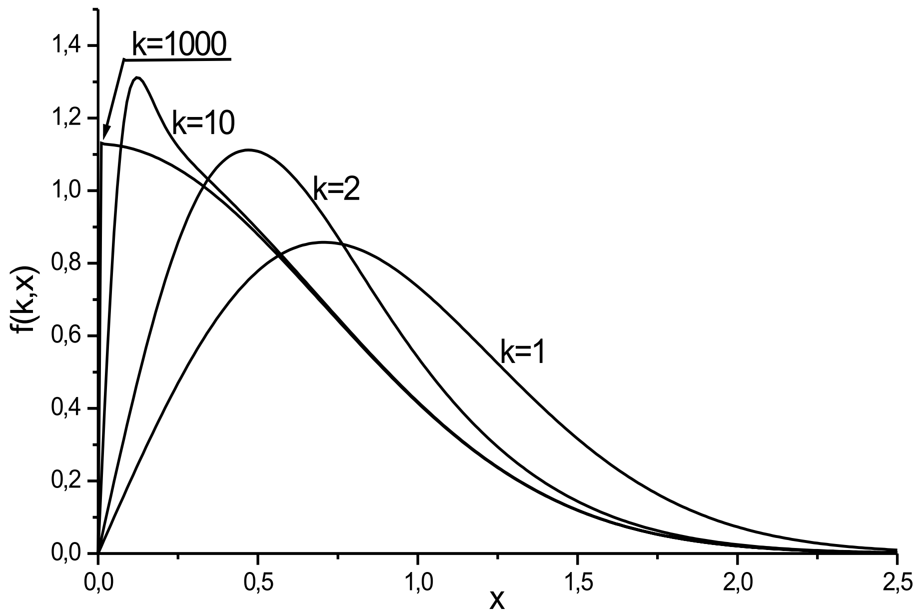

Figure 1: Distribution function $f(k,x)$, $x = p/\sigma_p$.

transverse momentum spread in a beam; c is the speed of light; γ is the Lorentz factor; $I_0(x)$ is the modified Bessel function.

At $k \to \infty$ the distribution function approaches the form corresponding to the one-dimensional collision case (see Fig.1) [4]:

$$f(p)dp = \frac{2}{\sqrt{\pi}\sigma_p} \exp\left(-\frac{p^2}{\sigma_p^2}\right) dp, \quad (p > 0).$$

At $k \to 1$ (the strong betatron coupling case) the distribution becomes the two-dimensional Maxwell one:

$$f(p) \propto p \cdot \exp\left(-p^2/\sigma_p\right).$$

Since the Møller cross section is proportional to $1/\nu^4$, the shift of the distribution function maximum to larger p's at smaller k's must affect an intensity of IBS. The use of the modified distribution function changes the forms of universal characteristic functions which describe the diffusion rate and the losses rate in the theory of IBS [6].

The determinative process in the integrated Touschek effect is the multiple scattering provided the latter significantly contributes to the energy diffusion in comparison with the synchrotron radiation (SR). Losses of particles (beam lifetime) due to single scattering depend on the steady beam dimensions determined by the total (SR + IBS) diffusion rate, radiative damping and betatron coupling. For the storage ring configuration in DAFNE SRFF experiment it would suffice to describe the betatron coupling in the " weak coupling" terms. Thus the beam cross section tilt is mostly negligible and the basic parameter is the ratio between the vertical ($\mathcal{E}_Y$) and horizontal emittances($\mathcal{E}_X$): $æ = \mathcal{E}_Y/\mathcal{E}_X$. Generally, the contribution from the vertical dispersion to the vertical size and angular spread also must be taken into account, but in our estimations it is enough to vary only the æ parameter.

Let denote

$$u = (\sigma_\gamma/\gamma)^2 = u_Q + u_T$$

- the squared relative energy dispersion;

$$v = \mathcal{E}_X = v_Q + v_T$$

- the radial phase volume;

$$k \approx \sqrt{\frac{\beta_Y[v(1+\alpha_X^2) + \beta_X \eta_X'^2 u]}{\beta_X æ v(1+\alpha_Y^2)}};$$

$$\mathcal{H} = [\eta_X^2 + (\beta_X\eta_X' + \alpha_X\eta_X)^2]/\beta_X$$

- the function describing the excitation of radial betatron oscillation due to an instant change in a particle energy; σ_Z - the longitudinal beam size, β_Y, α_Y, β_X, α_X, η_X, η_X' are respectively the amplitude and dispersion functions.

The indexes Q and T mark the contributions respectively of synchrotron radiation (quantum diffusion) and of Touschek effect. Total diffusion coefficients in respect of the particle energy and the radial emittance are given through the following sums:

$$D_u = D_u^Q + D_u^T,$$

$$D_v = D_v^Q + D_v^T,$$

where D_u^Q and D_v^Q are determined, for example, in [5]. The Touschek diffusion coefficients may be written as

$$D_u^T = \frac{Nr_0^2 c}{16\pi\gamma^3}\left\langle \frac{B(k,\chi_m)}{\sigma_X\sigma_Y\sigma_L\sigma_p} \right\rangle,$$

$$D_v^T = \frac{Nr_0^2 c}{16\pi\gamma^3}\left\langle \frac{B(k,\chi_m)\mathcal{H}}{\sigma_X\sigma_Y\sigma_L\sigma_p} \right\rangle.$$

Here angle brackets mean the averaging over the machine azimuth (ϑ) that implies variability of the beam length among other azimuth-dependent parameters; N is the number of particles in a bunch. The quantity $B(k,\chi_m)$ is a modified diffusion rate function [6] which in contrast to the one-dimensional collision theory [5] depends on the coupling parameter k:

$$B(k,\chi_m) = \sqrt{\pi}k\int\limits_{\chi_m}^{\infty}\sqrt{\frac{1}{\chi}}\cdot\ln\left(\frac{\chi}{\chi_m}\right)\cdot S(\chi,k)d\chi; \tag{2}$$

r_0 is the classical electron radius; $\chi_m = (p_m/\sigma_p)^2$; $p_m = p_0\sqrt{r_0/b_{max}}$, the classical lower limit of momentum transfer; b_{max} is the maximal scale of the impact parameter in CMS.

We determine the latter as usually through the average particle density in the co-moving system:

$$b_{max} = \left(\frac{\gamma V}{N}\right)^{1/3},$$

$V = 8\pi^{3/2}\sigma_X\sigma_Y\sigma_L$ is the bunch volume in the laboratory system.

The steady-state values of u and v are determined from the system of transcendental equations

$$u = u_Q + \frac{\tau_E}{2} D_u^T, \tag{3.1}$$

$$v = v_Q + \frac{\tau_X}{2} D_v^T, \tag{3.2}$$

τ_E and τ_X are respectively the damping times for synchrotron and radial betatron oscillations. The quantities u_Q and v_Q from SR contribution are used as input values in solving this system

The loss rate (the inverse beam lifetime) due to Touschek processes may be found from the formula:

$$\frac{1}{\tau} = 2\sqrt{\pi} r_0^2 m^3 c^4 N \left\langle \frac{C(k,\varepsilon)}{\sigma_p A_p^2 V} \right\rangle. \tag{4}$$

Here

$$C(k,\varepsilon) = \sqrt{\pi} k\varepsilon \int_{\varepsilon}^{\infty} \chi^{-\frac{3}{2}} \left[\frac{\chi}{\varepsilon} - \frac{1}{2}\ln\frac{\chi}{\varepsilon} - 1\right] \cdot S(\chi,k) d\chi,$$

the modified "loss function" which depends on the parameters k and $\varepsilon = A_p^2/\gamma\sigma_p^2$ with A_p, the "energy aperture" limiting the deviation of the longitudinal momentum from the equilibrium value.

3 Energy aperture calculation

To find the energy aperture distribution over the machine azimuth we apply 6D Particle Tracking for nonlinear dynamics simulation based on Acceleraticum Code [5]. At a given azimuth, a particle starts with $\Delta p/p \neq 0$ and infinitesimal seed deviations from the equilibrium orbit. Maximal $\Delta p/p$ is found that does not yet result in particle loss. Thus, the energy aperture $A_p = \min(A_{RF}, A_{DA})$ is automatically determined between A_{RF}, RF separatrix size, and A_{DA}, the dynamic aperture limit. As a result, we obtain the azimuth-dependent energy aperture A_p which determines the IBS particle loss rate.

4 Results of Toushek effects calculation

The concept of DAFNE SRFF Experiment includes an application of the Superconducting RF 1.3 GHz Cavity in addition to the normal one (368 MHz) operating in the Weak RF Focusing regime. Below we present the results of Touschek effects calculation at the following values of Superconducting RF Cavity Voltage: $U_{RF} = 0,\ 1,\ 4$ and 8 MegaVolts. Besides, we varied the coupling parameter æ (0.001, 0.01, 0.06, 0.1) as well as the beam current per one bunch ($I_b = 0.1, 1, 4, 8, 10$ mA). Three variants of the DAFNE magnetic lattice (A, B and C) were under considerations. They differ in the character of the $R_{56}(s)$ dependence (see Fig.2). In the case A the function R_{56} is monotonic in s. In two other cases $R_{56}(s)$ is not monotonic and even oscillates in its sign that gives notably smaller values of the synchrotron tune ν_s in comparison with the A case ($\nu_s(A) = 0.2433$, $\nu_s(B) = 0.1544$, $\nu_s(C) = 0.0770$). Beam length variation along the ring is presented in Fig.3 for the A case. Most deep beam length modulation with a factor 2 takes place at the maximal SRFF Cavity voltage from the set above. The behavior of the corresponding Energy Aperture (EA) with the azimuth is shown in Fig.4. The averaged EA at 1 MV SRFF Cavity voltage is about 20% larger as compared with the Weak RF focusing regime level. In a given case, it may mean that EA expands due to increase of the RF bucket size. But with further increase in the SRFF voltage, EA starts to oscillate strongly with the azimuth. At $U_{RF} = 8$ MV EA drops down in minimum even below its level at $U_{RF} = 0$. Evidenty, this can be explained by strengthening of synchro-betatron resonances influence. Among the versions under consideration, the C one has the lowest average level of EA (see Fig.5).

Calculations using the method, described in section 2, showed the gains of the energy spread and the horizontal emittance caused by Touschek effect to be not significant (see Fig.6 and 7). The life time (the A case) is plotted as a function of the coupling æ at 1 mA bunch current in Fig.8. To represent in more detail the behavior of particle losses in the typical values region æ ~ 0.01, the loss rate, or inverse beam life time, is shown as depending on the bunch current function in Fig.9. The most large beam life time among the versions A, B. and C of SRFF Experiment DAFNE is achieved in the B case - see Fig.10. On this point, A and B versions are close each to other, but C version greatly differs in a bad sense from A and B in accordance with the azimuthal dependence of EA in Fig.5.

5 Conclusions

1) At $U_{RF} = 8$ MV, the beam population $N \approx 10^{10}$ particles (about 5 mA bunch current), the coupling æ $= 0.01$, the beam life time can be about 10 minutes that opens opportunity for SRFF experiment from this side.
2)Taking into account the azimuthal variation of Energy Aperture and Beam Length is important for correct consideration of Touschek contribution. At $U_{RF} = 8$ MV the ratio of the maximal EA to the minimal one makes up about 3, while Loss Rate varies as squared EA.

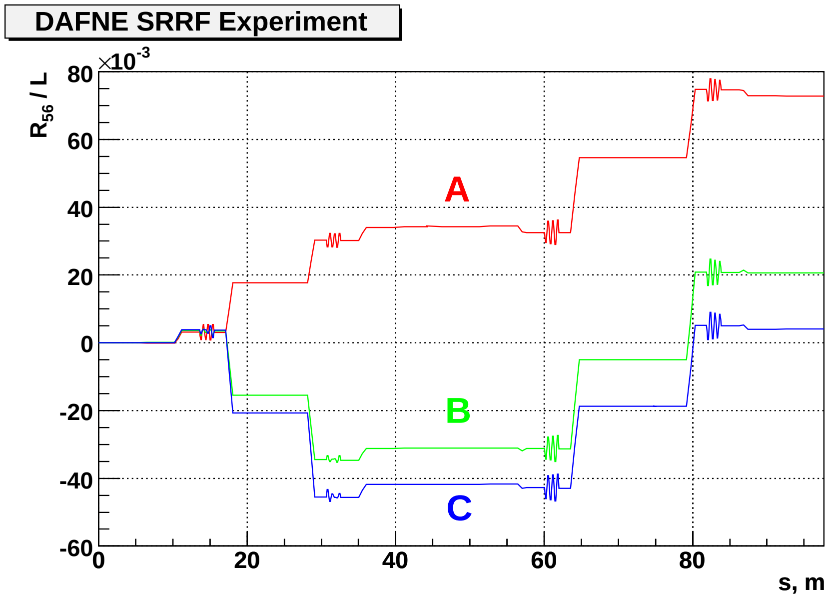

Figure 2: Function describing sensitivity of the longitudinal position of a particle to its energy deviation versus the azimuth in A, B, and C versions of SRFF DAFNE.

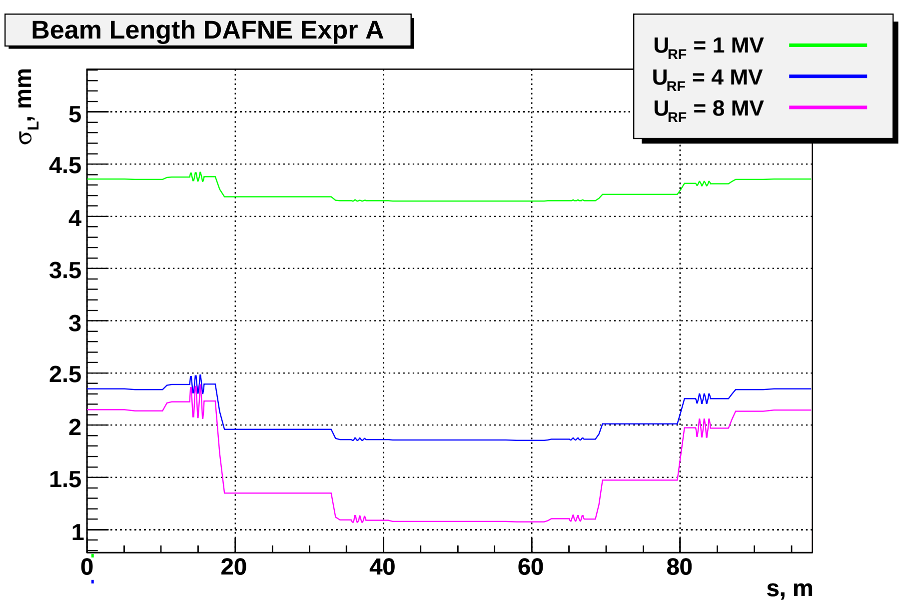

Figure 3: Beam Length vs. the azimuth in SRFF DAFNE (A).

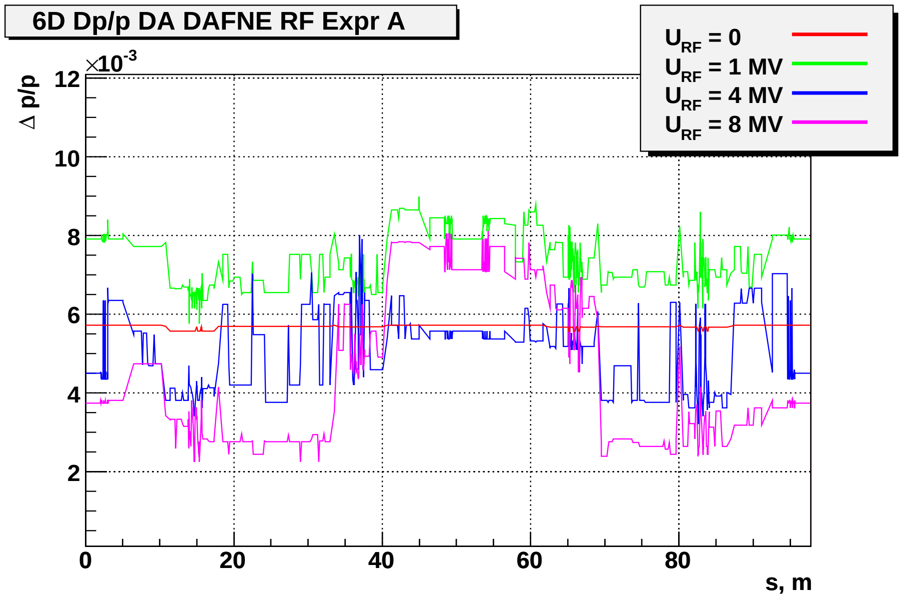

Figure 4: Energy Aperture versus the azimuth in SRFF DAFNE (A) at different U_{RF}.

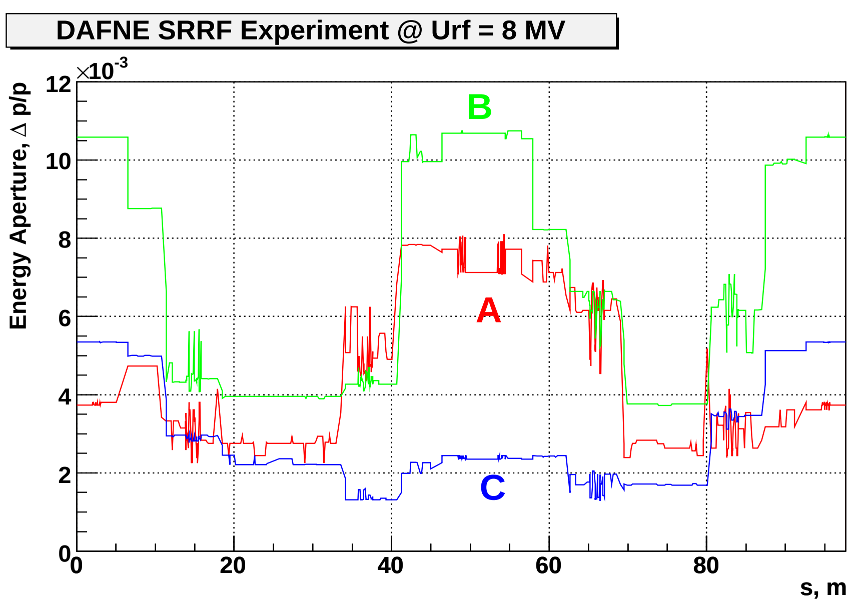

Figure 5: Energy Aperture versus the azimuth in versions A, B, and C at 8 MV.

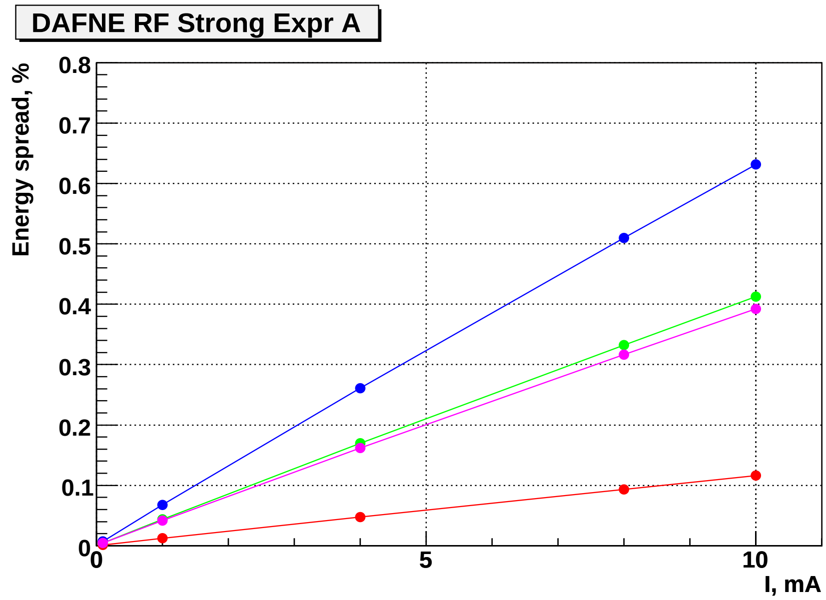

Figure 6: Relative gain in the energy spread due to IBS in SRFF DAFNE (A) vs. the bunch current at the coupling æ = 0.01 and different U_{RF} (a correspondence of the line colours to the voltage values is the same as in preceding figures).

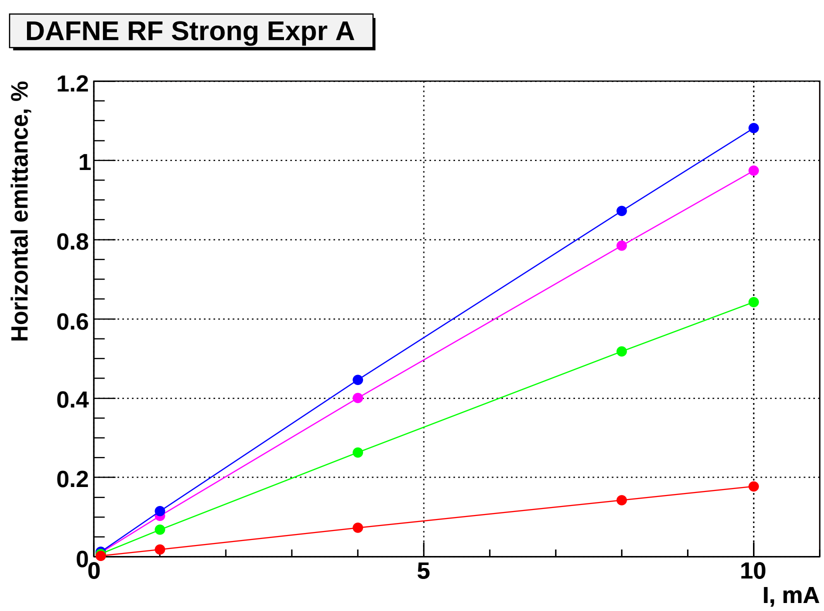

Figure 7: Relative emittance gain due to IBS in SRFF DAFNE (A) vs. the bunch current at the coupling æ = 0.01 and different U_{RF}.

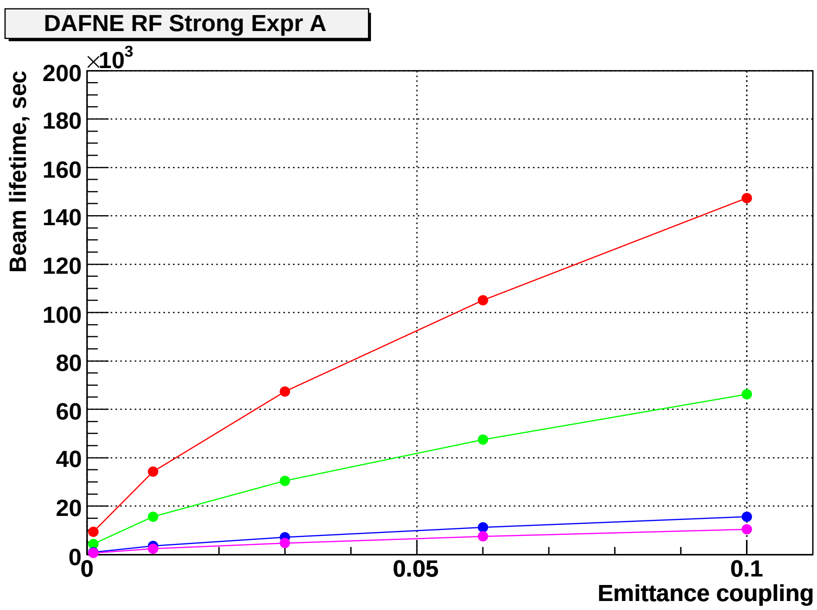

Figure 8: Beam Life Time vs. Coupling for SRFF DAFNE (A) at 1mA Bunch Current and different U_{RF}.

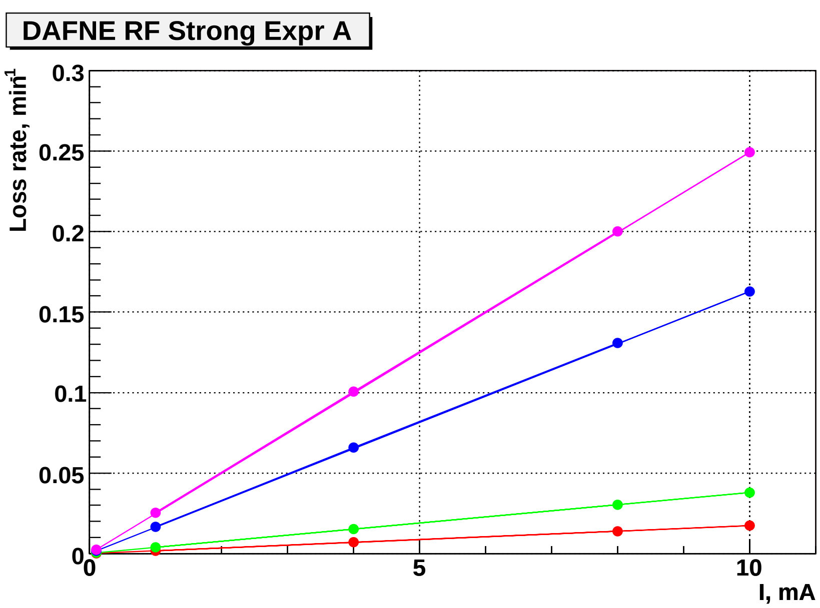

Figure 9: Loss Rate due to IBS for SRFF DAFNE (A) vs. Bunch Current at æ = 0.01 and U_{RF}.

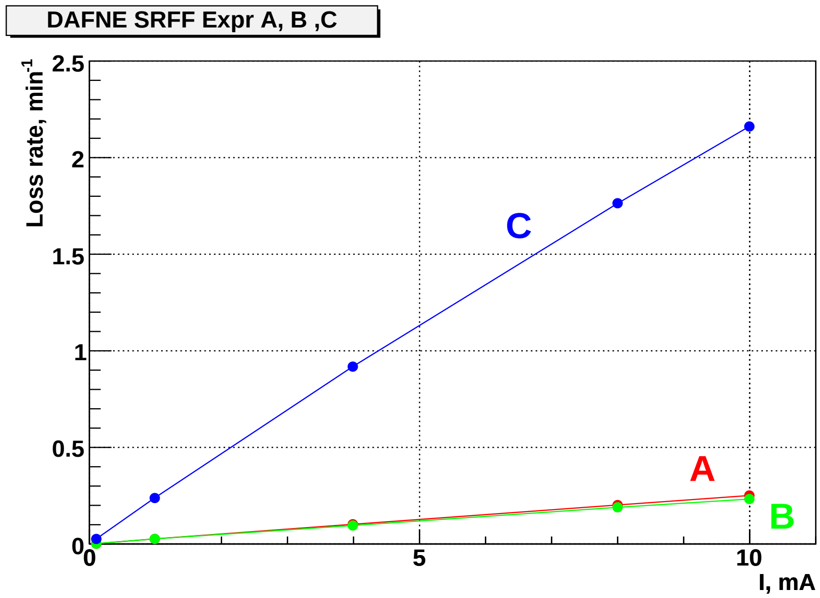

Figure 10: Comparison of Loss Rate vs. Bunch Current dependences for A, B and C versions at æ = 0.01.

3) For SRRF DAFNE versions considered the influence of IBS on the gains in Beam Emittance and Energy Spread is negligible ($< 1\%$).

6 Acknowledgements

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