DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

Frascati, March 3, 1999

Note: **G-52**

SOME RESULTS OF LIE TRANSFORM ON SEXTUPOLE HAMILTONIAN

R. Huang

The trigonometric series of the sextupole Hamiltonian suitable for asymmetric lattice is given. The results of Lie transformation to the first order of two dimensional space, and to the second order of one dimensional space are also given. Some results of the estimated dynamic aperture with DAΦNE parameters are compared with the results given by tracking.

1. Introduction

"Particle tracking" has given good results of the dynamic aperture in DAΦNE[1]. However, in order to be prepared for any necessary further dynamics investigation, experimentally or theoretically, we try to do some estimate from other approach with more "physics" underlined. That's the "Hamiltonian method", which has been intensively studied by E. Levichev, et al. on VEPP-4M and SIBERIA2[2,3,4]. During their analyses, they started their work with the Hamiltonian of sextupoles in the form of cosine series. The formulas of the one dimensional Lie transform up to second order were also given in their publications[2,3].

However we will explain in the following that for an accelerator with asymmetry lattice, such as DAFNE with two different interaction regions, the trigonometric series of sextupole Hamiltonian should include both the sine and cosine series. Therefore the Lie transformation for the Hamiltonian suitable for asymmetry lattice will be done in this note.

The dynamic apertures of different DAΦNE configurations are estimated with the formulas we got. The estimated apertures are generally smaller than those given by tracking while nearly half of them are close to the tracking results.

2. Sextupole Hamiltonian and Lie transform

2.1 Sextupole Hamiltonian

After appropriate canonical transformation, K.Y. Ng got the sextupole Hamiltonian[5]:

$$
\begin{aligned}
H = \nu_x J_x + \nu_z J_z + (2J_x)^{3/2} \beta_0^{1/2} \sum_m \left(A_{3m} \sin q_{3m} + 3A_{1m} \sin q_{1m} \right) \\
-(2J_x)^{1/2} (2J_z) \beta_0^{1/2} \sum_m \left(2B_{1m} \sin p_{1m} + B_{+m} \sin p_{+m} + B_{-m} \sin p_{-m} \right)
\end{aligned}
\tag{1}
$$

where $q_{1m,3m}$ and $P_{1m,3m}$ are related not only with the harmonic number along "θ" coordinate, but also the distribution of the sextupoles. According to the definition of each symbol in eq.(1)[5], we can get the formula of sextupole Hamiltonian:

$$
\begin{aligned}
H = {} & \nu_x J_x + \nu_y J_y \\
& + (2J_x)^{3/2} \sum_m \{ 3A_{1m} \cos(\phi_x - m\theta) - 3Z_{1m} \sin(\phi_x - m\theta) \\
& \qquad + A_{3m} \cos(3\phi_x - m\theta) - Z_{3m} \sin(3\phi_x - m\theta) \} \\
& -3(2J_x)^{1/2} (2J_z) \sum_m \{ 2B_{1m} \cos(\phi_x - m\theta) - 2Y_{1m} \sin(\phi_x - m\theta) \\
& \qquad + B_{+m} \cos(\phi_x - m\theta) - Y_{+m} \sin(\phi_x - m\theta) \\
& \qquad + B_{-m} \cos(\phi_x - m\theta) - Y_{-m} \sin(\phi_x - m\theta) \}
\end{aligned}
\tag{2}
$$

where:

$$
\begin{aligned}
A_{jm} &= \frac{1}{48\pi} \sum_k \left[\left(\beta_x^{3/2} K_s \right) \cos\left(j(\psi_x - \upsilon_x \theta) + m\theta \right) \right]_k \\
Z_{jm} &= \frac{1}{48\pi} \sum_k \left[\left(\beta_x^{3/2} K_s \right) \sin\left(j(\psi_x - \upsilon_x \theta) + m\theta \right) \right]_k \\
B_{1m} &= \frac{1}{48\pi} \sum_k \left[\left(\beta_x^{1/2} \beta_z K_s \right) \cos\left(\psi_x - \upsilon_x \theta + m\theta \right) \right]_k \\
Y_{1m} &= \frac{1}{48\pi} \sum_k \left[\left(\beta_x^{1/2} \beta_z K_s \right) \sin\left(\psi_x - \upsilon_x \theta + m\theta \right) \right]_k \\
B_{\pm m} &= \frac{1}{48\pi} \sum_k \left[\left(\beta_x^{1/2} \beta_z K_s \right) \cos\left(\psi_\pm - \upsilon_\pm \theta + m\theta \right) \right]_k \\
Y_{\pm m} &= \frac{1}{48\pi} \sum_k \left[\left(\beta_x^{1/2} \beta_z K_s \right) \sin\left(\psi_\pm - \upsilon_\pm \theta + m\theta \right) \right]_k
\end{aligned}
\tag{3}
$$

and $K_s = \frac{L}{B\rho} B_z'' \ , \ \psi_u = \int_s \frac{ds'}{\beta_u} \ , \quad \psi_\pm = \psi_x \pm 2\psi_z \ , \quad \nu_\pm = \nu_x \pm 2\nu_z$, j=1,3

Here we keep the definition of all symbols same with those given by E. Levichev et al.[2,3,4] so that they will be slightly different from those given by K.Y. Ng[5].
It can be seen from eqs.(2) and (3) that for symmetric lattice, which are the situations of lots of light sources, the Hamiltonian will be same as that given by E. Levichev and V. Sajaev because all the sums of the sine series in eq.(3) will be zero. However for asymmetric lattice, such as the situation of DAΦNE with two different interaction regions, neither the sine series nor the cosine series could be neglected.

Both the Hamiltonian formulas given in eq.(1) and eq.(2) are suitable for Lie transformation. However in order to compare our results directly with those got by E. Levichev and V. Sajaev, we will start our work form eq.(2).

2.2 Lie transformation results

The Lie transformation is used for the system whose Hamiltonian is the sum of h_0, for which the trajectory is known, and some small perturbations h_n. There are some detail descriptions of such application on accelerator physics[4,6,7,8]. We will do the transformation here following the steps of the so-called Deprit perturbation theory[6,7], the same steps that E. Levichev and V. Sajaev have already used. However, here we will give some more general results than theirs so that our results could be used for some estimation on DAΦNE with asymmetric lattice.

In Deprit perturbation technique, the old Hamiltonian H, the new Hamiltonian $\overline{H}$ and the generating function w are expanded as the power series:

$$H = \sum_{n=0}^{\infty} \varepsilon^n H_n \ , \qquad \overline{H} = \sum_{n=0}^{\infty} \varepsilon^n \overline{H}_n \ , \qquad w = \sum_{n=0}^{\infty} \varepsilon^n w_{n+1} \tag{4}$$

and the Lie transformation operator:

$$\hat{T} = \sum_{n=0}^{\infty} \varepsilon^n \hat{T}_n$$

In our case, $\varepsilon \sim \sqrt{4}$. The generating function of such transformation can be found step by step from:

$$\hat{D}_0 w_n = n(\overline{H}_n - H_n) - \sum_{m=1}^{n-1} \left(\hat{L}_{n-m} \overline{H}_m + m \hat{T}_{n-m}^{-1} H_m \right) \tag{5}$$

where $\overline{D}_0 = \partial / \partial\theta + [\ , H_0]$ is the derivative operator along the trajectory of the unperturbed system, $\hat{L}_n = [w_n, \ ]$ is the Poisson bracket operator, $\hat{T}_n^{-1}$ is the reverse operator of $\hat{T}_n$. The new Hamiltonian will be chosen in such values to cancel the secular terms during the integration of the RHS of eq.(5).

.

In this paper we will give the results for two cases: (a) in two dimensional space transform up to the first order of perturbation; (b) in one dimensional space transform up to the second order. Both the sine series and the cosine series will be included in the Hamiltonian transformation. Because the detail procedures of the transformation can be found somewhere else[4,6,7,81, only the brief results of our derivation will be given in the following, for two situations:

A. two dimensions, first order transformation

$$
\begin{aligned}
&\overline{H}_0 = \nu_x J_x + \nu_z J_z \\
&\overline{H}_1 = 0
\end{aligned}
$$

$$
\begin{aligned}
w_1 = &-(2J_x)^{3/2}\sum_m\left\{\frac{3A_{1m}}{\nu_x - m}\sin(\phi_x - m\theta) + \frac{A_{3m}}{3\nu_x - m}\sin(3\phi_x - m\theta)\right. \\
&\left.+\frac{3Z_{1m}}{\nu_x - m}\cos(\phi_x - m\theta) + \frac{Z_{3m}}{3\nu_x - m}\cos(3\phi_x - m\theta)\right\} \\
&+3(2J_x)^{1/2}(2J_z)\sum_m\left\{\frac{2B_{1m}}{\nu_x - m}\sin(\phi_x - m\theta) + \frac{B_{+m}}{\nu_+ - m}\sin(\phi_+ - m\theta)\right. \\
&+\frac{B_{-m}}{\nu_- - m}\sin(\phi_- - m\theta) + \frac{2Y_{1m}}{\nu_x - m}\cos(\phi_x - m\theta) \\
&\left.+\frac{Y_{+m}}{\nu_+ - m}\cos(\phi_+ - m\theta) + \frac{Y_{-m}}{\nu_- - m}\cos(\phi_- - m\theta)\right\}
\end{aligned} \tag{6}
$$

and the relationship between new momenta and old ones:

$$
\begin{aligned}
\bar{J}_x = J_x + &J_x^{3/2}\left\{\sum_m a^{(1)}(1,0,m)\cos(\phi_x - m\theta) + \sum_m a^{(1)}(3,0,m)\cos(3\phi_x - m\theta)\right. \\
&\left.+\sum_m c^{(1)}(1,0,m)\sin(\phi_x - m\theta) + \sum_m c^{(1)}(3,0,m)\sin(3\phi_x - m\theta)\right\} \\
&-J_x^{1/2}J_z\left\{\sum_m b^{(1)}(1,0,m)\cos(\phi_x - m\theta) + \sum_m b^{(1)}(1,2,m)\cos(\phi_+ - m\theta)\right. \\
&+\sum_m b^{(1)}(1,-2,m)\cos(\phi_- - m\theta) + \sum_m d^{(1)}(1,0,m)\sin(\phi_x - m\theta) \\
&\left.+\sum_m d^{(1)}(1,2,m)\sin(\phi_+ - m\theta) + \sum_m d^{(1)}(1,-2,m)\sin(\phi_- - m\theta)\right\}
\end{aligned} \tag{7}
$$

$$\bar{J}_z = J_z - J_x^{1/2} J_z \left\{ 2\sum_m b^{(1)}(1,2,m)\cos(\phi_+ - m\theta) - 2\sum_m b^{(1)}(1,-2,m)\cos(\phi_- - m\theta) \right.$$
$$\left. + \sum_m 2d^{(1)}(1,2,m)\sin(\phi_+ - m\theta) - \sum_m 2d^{(1)}(1,-2,m)\sin(\phi_- - m\theta) \right\} \tag{8}$$

where:

$$a^{(1)}(1,0,m) = 6\sqrt{2}\frac{A_{1m}}{v_x - m}, \quad a^{(1)}(3,0,m) = 6\sqrt{2}\frac{A_{3m}}{3v_x - m}$$
$$c^{(1)}(1,0,m) = -6\sqrt{2}\frac{Z_{1m}}{v_x - m}, \quad c^{(1)}(3,0,m) = -6\sqrt{2}\frac{Z_{3m}}{3v_x - m}$$
$$b^{(1)}(1,0,m) = 12\sqrt{2}\frac{B_{1m}}{v_x - m}, \quad b^{(1)}(1,\pm 2,m) = 6\sqrt{2}\frac{B_{\pm m}}{v_\pm - m} \tag{9}$$
$$d^{(1)}(1,0,m) = -12\sqrt{2}\frac{Y_{1m}}{v_1 - m}, \quad d^{(1)}(1,\pm 2,m) = -6\sqrt{2}\frac{Y_{\pm m}}{v_\pm - m}$$

We keep the terminology same as those given by E. Levichev and V. Sajaev in order to verify our results by a simple comparison to theirs.

B. One dimensional, up to second order transform

$$\bar{H}_0 = v_x \bar{J}_x$$
$$\bar{H}_1 = 0$$

$$\bar{H}_2 = -18\bar{J}_x^2 \sum_m \left(\frac{3A_{1m}^2}{v_x - m} + \frac{A_{3m}^2}{3v_x - m} + \frac{3Z_{1m}^2}{v_x - m} + \frac{Z_{3m}^2}{3v_x - m} \right)$$

$$w_2 = 36J_x^2 \left\{ \sum_m^{m\neq 0} \sum_l \left(\frac{3A_{1l}A_{1m+l}}{v_x - l} + \frac{A_{3l}A_{3m+l}}{3v_x - l} + \frac{3Z_{1l}Z_{1m+l}}{v_x - l} + \frac{Z_{3l}Z_{3m+l}}{3v_x - l} \right) \frac{\sin(m\theta)}{m} \right.$$
$$- \sum_m^{m\neq 0} \sum_l \left[\frac{3(2v_x - m - 2l)A_{1l}Z_{1m+l}}{(v_x - l)(v_x - m - l)} + \frac{(6v_x - m - 2l)A_{3l}Z_{3m+l}}{(3v_x - l)(3v_x - m - l)} \right] \frac{\cos(m\theta)}{m}$$
$$+ \sum_m \sum_l \frac{2(4v_x - m - 2l)}{(v_x - l)(3v_x - m - l)} \left[(A_{1l}A_{3m+l} + Z_{1l}Z_{3m+l}) \frac{\sin(2\phi_x - m\theta)}{2v_x - m} \right.$$
$$\left. + (A_{1l}Z_{3m+l} - Z_{1l}A_{3m+l}) \frac{\cos(2\phi_x - m\theta)}{2v_x - m} \right] \tag{10}$$
$$+ \sum_m \sum_l \frac{(2v_x - m + 2l)}{(v_x - l)(3v_x - m + l)} \left[(Z_{1l}Z_{3m-l} - A_{1l}A_{3m-l}) \frac{\sin(4\phi_x - m\theta)}{4v_x - m} \right.$$
$$\left. \left. - (A_{1l}Z_{3m-l} + Z_{1l}A_{3m-l}) \frac{\cos(4\phi_x - m\theta)}{4v_x - m} \right] \right\}$$

and the relationship between new momentum and old one:

$$\begin{aligned}\bar{J}_x = J_x + J_x^{3/2}\Big\{&\sum_m a^{(1)}(1,0,m)\cos(\phi_x - m\theta) + \sum_m a^{(1)}(3,0,m)\cos(3\phi_x - m\theta)\\ &+\sum_m c^{(1)}(1,0,m)\sin(\phi_x - m\theta) + \sum_m c^{(1)}(3,0,m)\sin(3\phi_x - m\theta)\Big\}\\ &+J_x^2\Big\{\sum_m a^{(2)}(0,0,m)\cos(m\theta) + \sum_m a^{(2)}(2,0,m)\cos(2\phi_x - m\theta)\\ &+\sum_m a^{(2)}(4,0,m)\cos(4\phi_x - m\theta) + \sum_m c^{(2)}(0,0,m)\sin(m\theta)\\ &+\sum_m c^{(2)}(2,0,m)\sin(2\phi_x - m\theta) + \sum_m c^{(2)}(4,0,m)\sin(4\phi_x - m\theta)\Big\}\end{aligned} \tag{11}$$

where:

$$\begin{aligned}
a^{(2)}(0,0,m) &= 54\sum_l\left(\frac{A_{1l}A_{1m+l} + Z_{1l}Z_{1m+l}}{(\nu_x - l)(\nu_x - m - l)} + \frac{A_{3l}A_{3m+l} + Z_{3l}Z_{3m+l}}{(3\nu_x - l)(3\nu_x - m - l)}\right)\\
a^{(2)}(2,0,m) &= 72\sum_l\left(\frac{A_{1l}A_{3m+l} + Z_{1l}Z_{3m+l}}{(\nu_x - l)(3\nu_x - m - l)} \times \frac{2l - m}{2\nu_x - m}\right)\\
a^{(2)}(4,0,m) &= 36\sum_l\left(\frac{A_{1l}A_{3m-l} - Z_{1l}Z_{3m-l}}{(\nu_x - l)(3\nu_x - m + l)} \times \frac{4l - m}{4\nu_x - m}\right)\\
c^{(2)}(0,0,m) &= 108\sum_l\left(\frac{A_{1l}Z_{1m+l}}{(\nu_x - l)(\nu_x - m - l)} + \frac{A_{3l}Z_{3m+l}}{(3\nu_x - l)(3\nu_x - m - l)}\right)\\
c^{(2)}(2,0,m) &= 72\sum_l\left(\frac{Z_{1l}A_{3m+l} - A_{1l}Z_{3m+l}}{(\nu_x - l)(3\nu_x - m - l)} \times \frac{2l - m}{2\nu_x - m}\right)\\
c^{(2)}(4,0,m) &= -36\sum_l\left(\frac{A_{1l}Z_{3m-l} + Z_{1l}A_{3m-l}}{(\nu_x - l)(3\nu_x - m + l)} \times \frac{4l - m}{4\nu_x - m}\right)
\end{aligned} \tag{12}$$

When we set Z_{jm}=0,(j=1,3), we will see that all the $a^{(*)}(*,0,m)$ coefficients given here are same as those given by E. Levichev & V. Sajaev except $a^{(2)}(0,0,m)$ is different by a factor of 2. As for other coefficients, c*(*,*,m) are introduced by the lattice asymmetry effect, b*(*,*,m) are caused by two dimensional consideration, while d*(*,*,m) are caused by both of the above reasons.

3. Application for dynamic aperture estimation

Two ways have been mentioned by E. Levichev and V. Sajaev to estimate the dynamic aperture using the results of Lie transformation[2,3]: (a) Since the Lie transformed Hamiltonian is angle independent $(\partial\bar{H}/\partial\phi_{x,z}=0)$, the new momenta should be invariant of motion, therefore eqs.(7), (8), (11) can be written as $\bar{J}_{x,z}(J_{x,z},\phi_{x,z})=\bar{J}_{x,z}(J_{x,z}=0,\phi_{x,z}=0)$, and we can find the maximum $J_{x,z}$ beyond which there will be no solution of $J_{x,z}$ at some positions and that maximum $J_{x,z}$ will determine the dynamic aperture. (b) because normally only single-resonance or few harmonics dominate the total movement, E. Levichev and V. Sajaev used the single resonance approximation of the transformed Hamiltonian to get the dynamic aperture analytically from eqs. $\partial H/\partial\phi=0$, $\partial H/\partial J=0$ and the results are almost the same as those given by tracking[2].

In this note, we first try to estimate the dynamic aperture by finding the maximum existing contour of J_x of eq.(11) at a fixed θ position numerically (we choose θ=0 because so did E. Levichev and V. Sajaev in [2,3]). That means the calculation is up to the second order perturbation and using one dimensional approximation. For a given $J_x(\theta=0)$, the solution of J_x at 150 other points along the circle (ϕ =0-2π) are calculated with the Newton method. Since $xD = \sqrt{2\beta_x J_x}$, $\sigma_x = \sqrt{\beta_x\varepsilon}$ for zero coupling and in DAΦNE ε = 1.0E-6m·rad, there will be

$$x_D/\sigma_x = \sqrt{2.0\times10^6 J_x} \tag{13}$$

For calculation the dynamic aperture, we substitute the minimum value of J_x in outmost contour of $J_x(J_x\phi_x)$ into eq(13). When θ=0, eq.(11) becomes:

$$\begin{aligned}\bar{J}_x = J_x + J_x^{3/2}\left\{A_{110}\cos(\phi_x)+A_{130}\cos(3\phi_x)+C_{110}\sin(\phi_x)+C_{130}\sin(3\phi_x)\right\}\\ +J_x^2\left\{A_{200}+A_{220}\cos(2\phi_x)+A_{240}\cos(4\phi_x)+C_{220}\sin(2\phi_x)+C_{240}\sin(4\phi_x)\right\}\end{aligned} \tag{14}$$

where

$$\begin{aligned}
&A_{110}=\sum_m a^{(1)}(1,0,m)\,, && A_{130}=\sum_m a^{(1)}(3,0,m)\,,\\
&A_{200}=\sum_m a^{(2)}(0,0,m)\,, && A_{220}=\sum_m a^{(2)}(2,0,m)\,,\\
&A_{240}=\sum_m a^{(2)}(4,0,m)\,, && C_{110}=\sum_m c^{(1)}(1,0,m)\,,\\
&C_{130}=\sum_m c^{(1)}(3,0,m)\,, && C_{220}=\sum_m c^{(2)}(2,0,m)\,,\\
&C_{240}=\sum_m c^{(2)}(4,0,m)\,.
\end{aligned} \tag{15}$$

All the summations in eqs.(9), (12) and (15) will approach stable values when $|m|$ and $|l|$ become greater then several hundreds. For the calculation in this note, *m, l* are within a range from -1000 to 1000. The dynamic apertures for 5 different DAΦNE lattice configurations, two different working points for each configuration[1] are estimated and listed in Table 1. The subscript "A" means the asymmetry coefficients are included in the estimation. For asymmetric lattice, only the results including asymmetric coefficients are correct. The results without the asymmetric coefficients are kept in the table only for a comparison.

Table 1. Numerical results of e .(11)

conf*	1(a)	1(b)	2(a)	2(b)	3(a)	3(b)	4(a)	4(b)	5(a)	5(b)
A_{110}	-5.060	-4.744	2.4353	6.8063	13.101	-13.43	-14.74	7.1995	4.1724	-27.02
A_{130}	20.080	12.307	15.782	6.9838	22.575	26.475	12.761	11.626	11.048	23.256
A_{200}	321.98	130.81	191.56	71.469	173.57	661.25	285.58	141.18	105.41	951.83
A_{220}	-46.12	-1917	176.71	-1362	819.89	-827.7	-68.49	-3015	891.5	-184.7
A_{240}	135.69	425.50	141.35	104.75	-233.3	751.75	-187.7	-160.3	115.50	528.07
C_{110}					7.5985	-13.43	8.9130	-0.554	8.9045	-3.634
C_{130}					30.386	26.475	12.596	13.222	7.1534	9.3595
A_{200A}					1246.2	661.29	345.60	272.64	201.96	1029.0
A_{220A}					1876.7	-624.9	-73.19	-3723.	900.65	-1709.
A_{240A}					250.80	744.76	-115.8	453.99	206.54	706.90
C_{220}					-1813	-879.1	-230.8	1807.4	-159.2	-754.5
C_{240}					-513.5	448.35	-53.63	-821.1	-73.51	479.44
$J(*10^{-4})$	NL**	5.1382	NL	1.0335	1.9	1.1789	NL	1.0017	1.9	NL
$J_A(10^{-4})$					0.3664	0.5409	NL	0.3715	1.684	0.6500
D/σ_x	21.5	32.1	22.5	14.4	19.5	15.4	24.4	14.2	19.5	-----
D_A/σ_x					8.5	10.4	29.4	8.6	18.4	11.4

* The definition of the configurations are given in [1], for different lattice and working points.

** NL means no limit of JX has been found from eq.(l4).

The tracking results given in [1] show that the dynamic apertures of all the configurations are greater than $20\sigma_x$, except "3(a)" configuration where DA~17 σ_x. But in table 1, there are about half of the estimated dynamic apertures smaller than the tracking results. Currently we don't know why. However, except 3(a) and 4(b) configuration, all the estimated apertures are greater than the physical aperture, which is about 10 σ_x off coupling in horizontal plane.

It can be seen from Table 1 that for some cases we cannot find the limit of the J_x from eq.(14). That is because θ=0 limited the influence of A_{200} on J_x along the trajectory of the particle. If we use eq.(11) and $\theta=\phi/\nu_x$ (that means we find the solution of J_x along the unperturbed orbit), the limit of J_x for 1(a), 2(a), 4(a) configurations will be found and the dynamic aperture of these configurations in table 1 are given by such limits. We search the J_x value along the orbit for three circles (θ=0--6 π). However no limit of J_x can be found for 5(b) case if the asymmetry terms in the Hamiltonian are not considered.

From eq.(12) it can be seen that the value of the coefficients of eq.(14) represents how much influence the different harmonics will have on the momentum J_x. Effects of four harmonics: ν_x=m, $2\nu_x$=m, $3\nu_x$=m and $4\nu_x$=m, are included in eq.(11). For higher harmonic effect, one has to use the results of more higher order Lie transformation.

Table 1 shows that A_{130} is greater than A_{110} and C_{130} is greater than C_{130} for almost all the configurations. That means $3\nu_x$=m harmonics normally has stronger influence than ν_x=m harmonics. For l(b), 2(b), 3(b), 4(b) (ν_x=4.53) and 3(a), 5(a) cases (ν_x=5.09), all the A_{220} (or A_{220A}, C_{220}) values are quite big. That means $2\nu_x$=m harmonics have strong influences; For l(a), 2(a), 4(a) and 5(b) cases, the A_{220} (or A_{220A}) values are big, and it can be seen from eq.(l2) that the $3\nu_x$=m harmonics or a kind of "coupling" between the $3\nu_x$=m and ν_x=m harmonics will play an important role.

When J_x is big and close to the dynamic aperture limit, the numerical results of eq.(14) (or eq.(11)) shows that $J_x(\phi_x)$ is quite wavy along the trajectory, but when J_x is small and far away form the dynamic aperture limit, the $J_x(\phi_x)$ is quite smooth. So it is also possible to check how $J_x(\phi_x)$ behaviours when J_x close to the physical aperture limit. We do it in this way: Setting the maximum of J_x in the contour to be l0σ_x (the physical limit in DAΦNE) then to search the minimum value of $J_x(\phi_x)$ in the contour and find the aperture calculated from this minimum J_x, that gives the "total effect" of the sextuples and the physical limit. The results are shown in Table 2. In order to have a impression of the distortion of $J_x(\phi_x)$ plot when J_x is big, a set of maximum and minimum values (J_{1DA}, J_{2DA}) of J_x on the outmost contour we calculated are also listed (if for some cases there are no limit by eq.(14), we use the result of eq.(11) instead, i.e., searching the solution along particle trajectory). J_{1PH} was fixed to 5.0×10^{-5}m as the physical limit of $10\sigma_x$ and it was taken as the maximum value of the contour of which the minimum is J_{2PH}. D_t is the aperture calculated from J_{2PH} and we take it as the total effect of both sextupoles and physical limit. For 3(a) configuration, this estimate predicts a 30% decrease of aperture. In 3(a) configuration, not only the $2\nu_x$=m harmonics is strong (big A_{220A} value), but also $3\nu_x$=m harmonics (big A_{130} and C_{130} values).

Table 2.

conf*	1(a)	1(b)	2(a)	2(b)	3(a)	3(b)	4(a)	4(b)	5(a)	5(b)
$J_{1DA}(10^{-5})$	117.9	23.87	5034.	24.63	13.09	16.72	424.2	9.85	51.34	24.18
$J_{2DA}(10^{-5})$	23.2	10.21	25.29	10.33	3.66	5.41	43.11	3.72	16.84	4.26
$J_{1PH}(10^{-5})$	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
$J_{2PH}(10^{-5})$	3.71	3.85	3.95	3.93	2.44	3.22	3.50	2.80	3.75	2.90
D_t/σ_x	8.61	8.77	8.89	8.87	7.00	8.02	8.37	7.48	8.66	7.62

Summary

The trigonometric series of sextupole Hamiltonian suitable for Asymmetric lattice is given. The Lie transformation of (a) two dimensional first order and (b) one dimensional second order are also given. The dynamic apertures of 5 proposed DAΦNE configuration[1] (two work points of each configuration) have been estimated. Nearly half of the results are close to the tracked results while nearly half of the results are smaller than those given by tracking.

Acknowledgements

I would like to take this opportunity to thank Dr. Mikhail Zobov who first showed me the importance of the work of DAΦNE dynamic aperture and finally checked my mathematical deductions and my manuscript of this note. Many thanks to Dr. F. Sannibale, Dr. A. Ghigo and Dr. M. Serio for their continuos concerning of my work in Frascati, which is always a nice encouragement to me. Thanks to Dr. E. Levichev for sending me all of his relevant publications from Novosibirsk, Russia.

References

[1] M.E. Biagini, C. Biscari, S. Guiducci, "DAΦNE main rings lattice update", DAΦNE technical note L-22

[2] E. Levichev and V. Sajaev, "Nonlinear dynamic study of the SIBERIA-2 electron storage ring", AIP conf. Proc. 344, 160-169

[3] E. Levichev and V. Sajaev, "Nonlinear phase space study in a low-emittance light souce using harmonic approximation", Particle Accelerators 56, 161-180 (1997)

[4] V. Kiselev, E. Levichev, et al., "Dynamic aperture measurement at the VEPP-4M storage ring", Particle Accelerators 57,65-76 (1997)

[5] K.Y. Ng, Fermilab Report TM-1281(1984) or K.Y. Ng, KEK Report 87-11 (1987)

[6] L.Michelotti, "Moser-like transformations of using the Lie transform", Particle Accelerators 16 233-252 (1985)

[7] J.R. Cary, "Lie transform perturbation theory for Harmiltonian systems", Physics Reports 79 129-159 (1981)

[8] A.J. Dragt and J.M. Finn, "Lie series and invariant functions for analytic symplectic maps", J. Math. Phys. 17, 2215 (1976).