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A TIME DOMAIN SIMULATION CODE OF THE LONGITUDINAL MULTIBUNCH INSTABILITIES

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1. INTRODUCTION

The analytical study of the longitudinal dynamics of a beam interacting with an RF cavity is generally performed only in the case of small oscillations of equispaced equal bunches around their synchronous phase[1]. Furthermore a complete analytical treatment of the dynamics in the presence of a bunch-by-bunch feedback system to control longitudinal coupled bunch instabilities has not yet developed.

The purpose of this note is to describe the main features of a simulation code that executes a tracking of the longitudinal oscillations of the bunches for DAΦNE, with the aim of including the main phenomena affecting the beam dynamics (i.e. the bunch-by-bunch feedback, the effect of the HOMs, the synchrotron radiation).

2. THE SIMULATION CODE

The code can simulate different starting conditions. In order to study the transient beam loading effects on the HOMs that may be dominant in high intensity accelerators, such as DAΦNE, we have simulated the injection of a bunch, assuming all the others already stored in the ring.

We model each bunch as a single particle of given charge. Under this condition it is possible to simulate only the "rigid" oscillations that, however, are the most dangerous for the beam stability and, at the moment, the only ones that will be cured with the longitudinal feedback system.

Basically the core of the algorithm can be divided into three main parts:

1) propagation around the ring
2) feedback effect
3) beam-cavity interaction.

In Fig.1 we show a simplified flow chart of the code. The input data have been divided into three files: one for machine and cavity parameters, another for all the bunches, and the last one for the longitudinal feedback system.

We have chosen to track the synchrotron motion of all bunches, first along the ring and the feedback system, and then through the cavity. This choice reduces the computation time.

As output data, we obtain the phase oscillations of three selected bunches, the invariant amplitude of their motion (used in the evaluation of the instability growth rates), the maximum phase excursion of each bunch, and the kicker voltages.

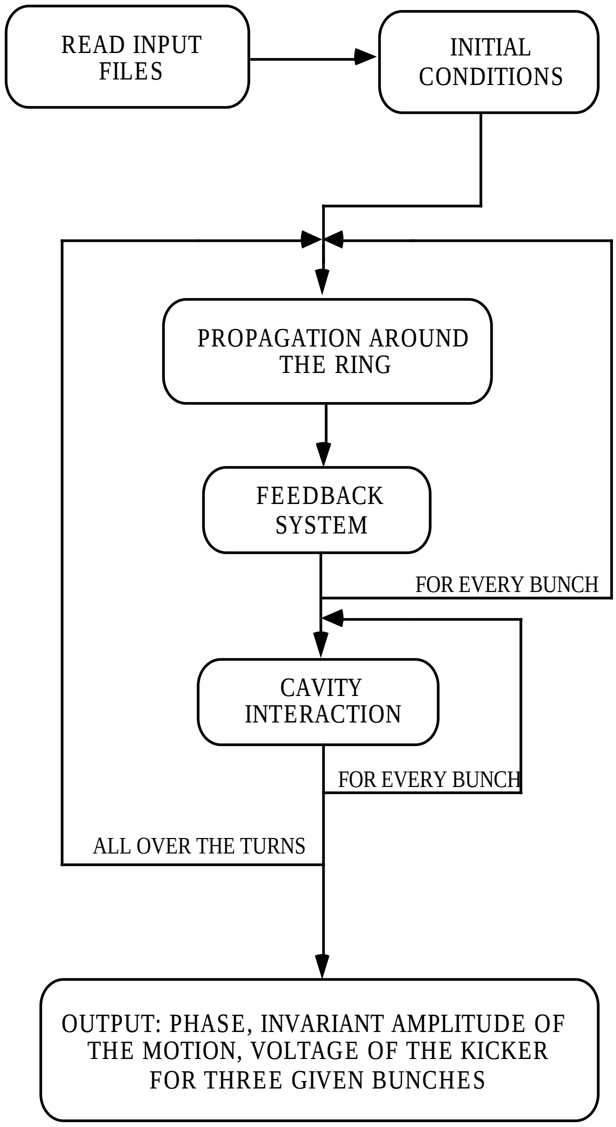

Fig. 1 - Flow chart.

2.1 Propagation around the ring

The quantities necessary to describe the motion of a single bunch in the machine, are the energy deviation ΔE with respect to the nominal energy E_o, and the phase $\Delta\varphi$ taken, in our case, with respect to the maximum of RF voltage.

In the propagation around the ring, each bunch loses energy due to the broad band impedance (U_{bb}) that does not depend on the energy of the bunch, and because of radiation effects (U_r) . For this last quantity we can work out the linear expression (as function of ΔE)[2]

$$U_r = U_o\left(1 + 2\frac{\Delta E}{E_o}\right) \tag{1}$$

where U_o is the energy lost by a synchronous particle.

It is therefore possible to correlate the quantities ΔE and $\Delta\varphi$ at the entrance of the feedback kicker with those outside the RF cavity. Since the kicker and the RF cavity are supposed dimensionless, the phase does not undergo any other change. The matrix that describes the motion in the machine can be written as

$$\begin{pmatrix}\Delta E\\ \Delta\varphi\end{pmatrix}_i = \begin{pmatrix}1-2\dfrac{U_o}{E_o} & 0\\ \dfrac{2\pi h\alpha_c}{E_o} & 1\end{pmatrix}\begin{pmatrix}\Delta E\\ \Delta\varphi\end{pmatrix}_o - \begin{pmatrix}U_o + U_{bb}\\ 0\end{pmatrix} \tag{2}$$

where 'i' means at the entrance of the kicker, 'o' means outside the RF cavity, h is the harmonic number, and α_c is the momentum compaction.

2.2 The feedback effect

Because of the high current stored, coupled bunches instabilities driven by the resistive impedance of the HOMs in the accelerating cavity have fast rise times.

A powerful longitudinal feedback[3,4] is necessary to damp the "rigid" oscillations and the injection transient. In the code subroutine shown in Fig. 2, all the devices are properly simulated.

The feedback system provides the "correction" energy to each bunch at every turn by means of a longitudinal kick. The phase-errror signal, detected by a longitudinal pick-up, is digitized and processed with a DSP digital filter which computes the correction signal K_k by the algorithm[3]:

$$K_k = G\sum_{i=1}^{N} c_i \Delta\varphi_{k-i} \tag{3}$$

where the filter coefficients c_i are computed in order to provide the 90° shift necessary to convert the phase error into the energy correction. The DSP output is amplified and sent through a digital-analog converter to a kicker amplifier.

In the input file concerning the feedback, it is possible to change the system configuration: we can vary the gains of the different devices, the number of coefficients, the coupling between consecutive bunches, the noise and the maximum kicker voltage. We can also simulate, by changing the coefficients c_i, different digital filters as: delay line, high and low pass, and resonant filters.

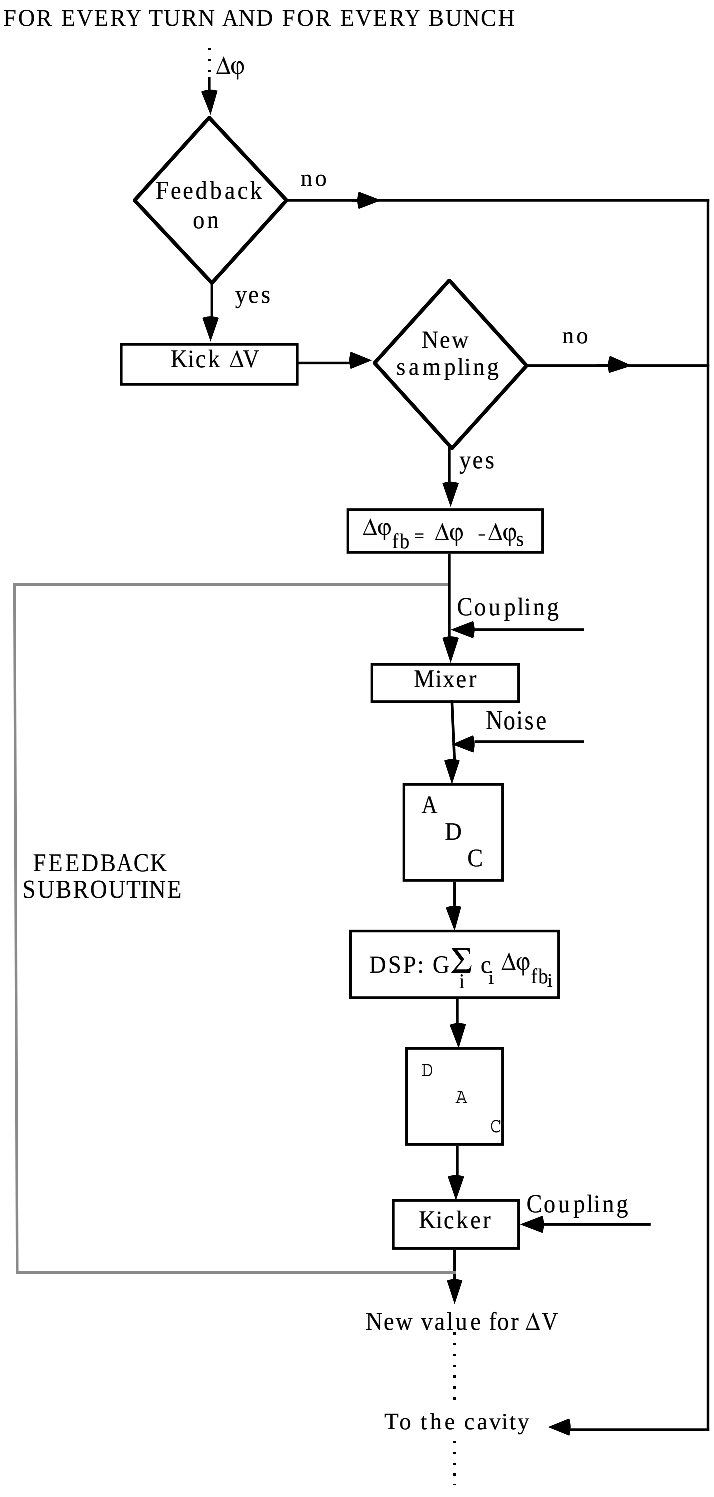

Fig. 2 - Feedback routine.

2.3 Beam-cavity interaction

The cavity is simulated as a series of parallel RLC circuits representing the HOMs. When a charge q_b crosses the cavity, it perturbs the total voltage. The induced voltage ΔV_m of each mode depends on the shunt resistance R_{sm} and the quality factor Q_m of that mode. We assume that the cavity-beam energy exchange occurs at a single point in the ring.

In order to take into account the bunch length, assuming a gaussian distribution, the shunt resistance is corrected by a factor[5]

$$\exp\left[-(\omega_m \sigma_t)^2\right] \tag{4}$$

where ω_m is the resonance angular frequency and σ_t the RMS bunch duration.

The RF generator voltage is given by

$$V_g = \hat{V}_g \cos(\Delta\varphi) \tag{5}$$

Since the beam loading in the fundamental cavity mode is very heavy, RF cavity feedback will be necessary to compensate it. In the simulations to date, we have assumed that the compensation is perfect, i.e. the fundamental mode of the wake field is not present in the cavity.

With the purpose of following the behavior of the induced wake voltage for each mode "m" in a matrix form, we use the conjugated variables $v_m(t)$ and the current in the inductance $i_m(t)$. Between the passage of two bunches, these quantities execute free oscillations that can be represented by the homogeneous solution of the differential equation of an RLC parallel circuit. We can therefore write[5]

$$\begin{pmatrix} v_m(t) \\ i_m(t) \end{pmatrix} = \exp(-\alpha_m t) \begin{pmatrix} \cos(\beta_m t) - \frac{\alpha_m}{\beta_m}\sin(\beta_m t) & -\frac{\omega_m R_{sm}}{\beta_m Q_m}\sin(\beta_m t) \\ \frac{\omega_m Q_m}{\beta_m R_{sm}}\sin(\beta_m t) & \cos(\beta_m t) + \frac{\alpha_m}{\beta_m}\sin(\beta_m t) \end{pmatrix} \begin{pmatrix} v_m(t_o) \\ i_m(t_o) \end{pmatrix} \tag{6}$$

where α_m is the cavity damping factor, β_m the natural angular frequency and $v_m(t_o)$ and $i_m(t_o)$ are the starting conditions.

When a bunch crosses the cavity, it is sufficient to increase $v_m(t)$ by the kick ΔV_m and continue the propagation. The total wake voltage seen by a single bunch is the sum all over the modes:

$$V_b = \sum_m v_m(t) \tag{7}$$

The total energy gained by the bunch in the RF cavity is therefore

$$E_c = e\left(\hat{V}\cos(\Delta\varphi) + V_b + \frac{1}{2}\sum_m \Delta V_m\right) \tag{8}$$

where the last term takes into account the fundamental theorem of beam loading: a bunch sees half of the wake voltage it induces during its passage.

3. COMPARISONS WITH ANALYTICAL RESULTS

3.1 Full coupling with a single resonator

For the computation of the growth rates, we have simulated the instability exciting only one mode of oscillation, leaving all the other modes unperturbed.

To do that, we have considered a single HOM with a resonant angular frequency equal to (full coupling)

$$\omega_r = p\omega_o + \omega_s \tag{9}$$

With the feedback off, and with all the bunches at the equilibrium phase, a small perturbation excites the selected mode of oscillation. We have then calculated the growth rates with an exponential fit over the invariant amplitude of the motion.

Fig.3 shows the results obtained varying the Q value of an HOM with p=500, R/Q=1, and I_{tot}=1.4A: the dots represent the values of the growth rates (α) given by the simulation code, the two curves are computed according to Laclare theory of the bunched beam coherent instabilities[1]: the 'a' curve is derived by calculating the HOM impedance at the unperturbed synchrotron frequency (that is the customary way of computing the rise time with codes such as BBI, ZAP), while the 'b' curve represents the more accurate result found solving the eigenvalues problem[6].

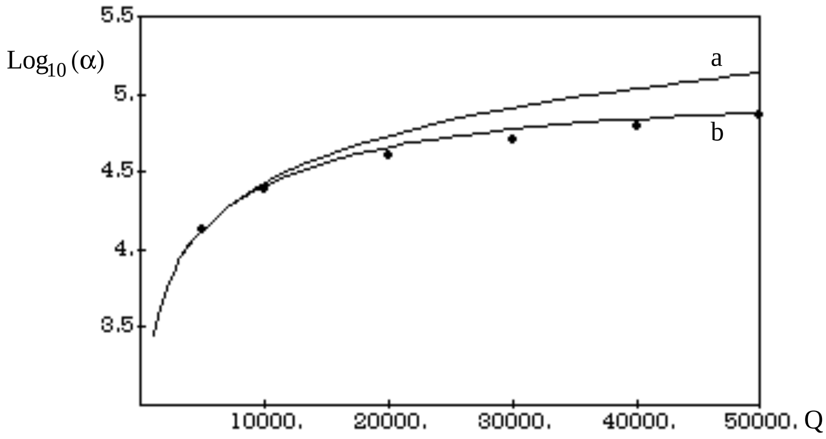

Fig. 3 - Growth rates for a single HOM versus Q.

3.2 Off resonance coupling

If we vary the resonant frequency of a single HOM (Q=1000, R/Q=1, p=500), in the neighbourhood of $p\omega_o+\omega_s$, we obtain a synchrotron frequency shift and a growth rate depending on the frequency of the HOM itself.

As in the full-coupling case, we have compared the growth rates and the frequency shifts obtained by the code, with those predicted by Laclare theory. Figures 4 and 5 show that also in this case the results are very satisfactory.

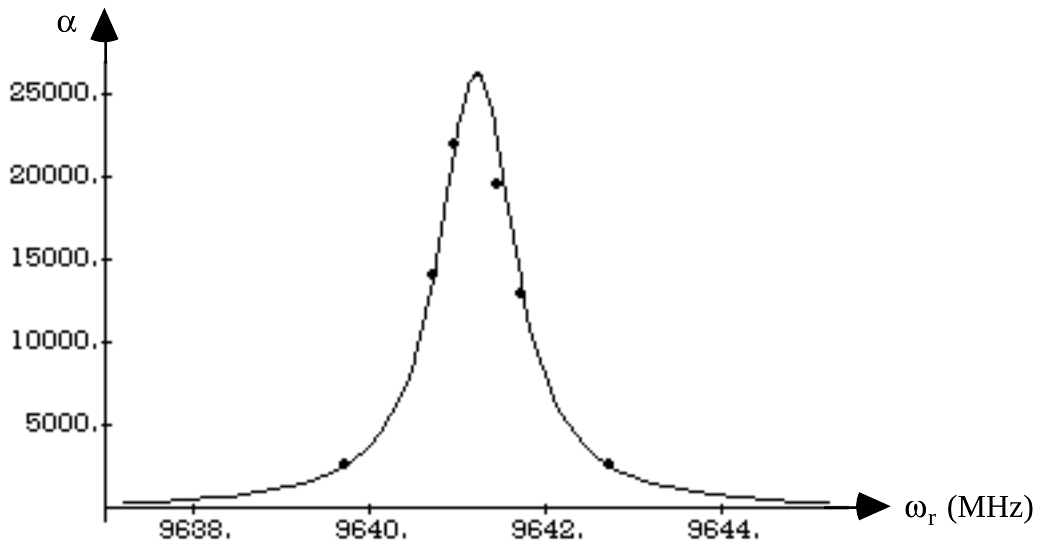

Fig. 4 - Growth rate of a single HOM versus ω_r.

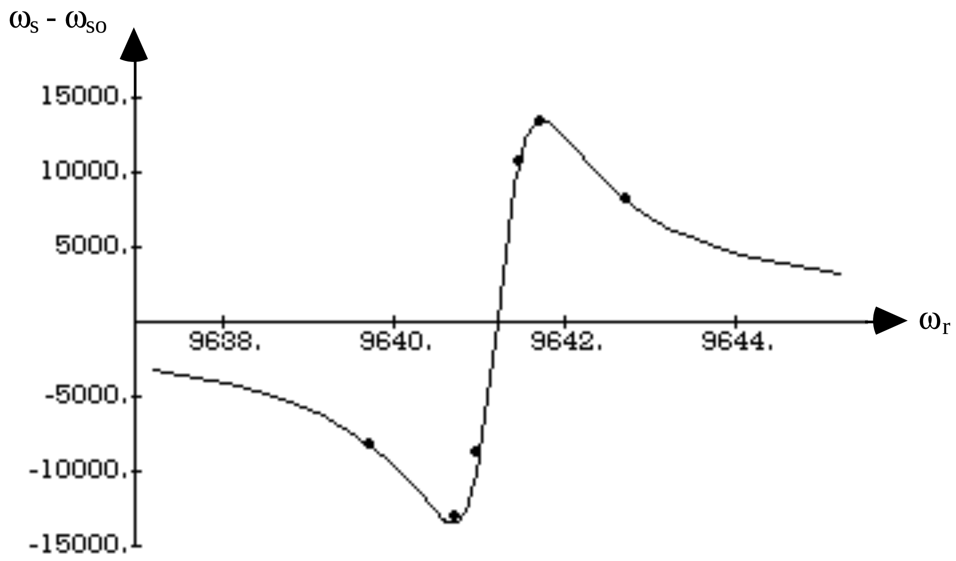

Fig. 5 - Synchrotron frequency shift of a single HOM versus ω_r.

3.3 Feedback damping time

Once we have determined all the parameters of the feedback system, namely the kicker voltage, the filter coefficients, and the internal gains, it is possible to "measure" the overall gain g (Volt/rad) with a code that gives the feedback response to a sinusoidal signal.

With the formula[3]

$$\frac{1}{\tau} = \frac{1}{2} f_{rf} \frac{\alpha_c}{E_o} \omega_o g \frac{1}{\omega_s} \qquad (10)$$

we can therefore compute the damping time of the feedback in the linear approximation.

This damping time has been compared with the damping time derived from simulating an off-energy beam, in absence of HOMs and radiation losses. Under this condition there is no coupling between the bunches and only the feedback is responsible of the damped oscillations.

The exponential fit over the invariant amplitude of the motion gives a damping time that, in the worst case, differs only of few per cent from that obtained with the equation (10).

4. APPLICATION TO DAΦNE

When several HOMs are present in the cavity, it is difficult to calculate analytically the rise time of all the possible modes of oscillation. There could be compensation between different HOMs, or, at the opposite, their effects could sum up. To compute exactly the growth rates with the Laclare theory, it should be necessary to solve a non-linear equations' system.

We have performed different simulations with 30 bunches, considering all the measured HOMs Q values (up to 2 GHz) of the cavity with nose-cones (waveguide loaded).

For lack of measured data, the R/Q values are those computed by URMEL code.

Since the frequency may vary during the machine operations, we have chosen the worst case: all the HOMs are in full coupling with the unstable sidebands of the beam spectrum.

We have first run the code with the feedback off, assuming 29 bunches at their synchronous phases, and simulating the injection of the 30^{th} with an initial error of 100 psec. This last bunch perturbs all the others that begin to oscillate with growing amplitude.

Tab.1 - HOMs values for the cavity with nose-cones.

Q_m	$R_m / Q_m (\Omega)$	ω_m (*10^6)	ω_m / ω_{RF}
180	4.033	4415.806	1.9804
6800	0.002	5977.642	2.5834
1000	0.264	6768.201	2.9251
650	0.190	6999.584	3.0251
1000	0.008	7828.707	3.3834
500	0.227	8156.500	3.5251
160	0.302	8657.830	3.7418
1100	0.428	9004.905	3.8918
360	1.400	9178.442	3.9668
1260	1.011	9583.363	4.1418
465	0.143	9737.618	4.2084
400	0.190	10219.666	4.4168
1960	1.026	10470.331	4.5251
300	0.804	11029.507	4.7668
245	3.284	11145.199	4.8168
650	0.345	11357.300	4.9084
814	0.002	11762.220	5.0834
2150	1.531	12012.885	5.1918
1580	0.022	12456.370	5.3834
570	0.910	12552.779	5.4251

The phase oscillations of the injected bunch are shown in Fig. 6 over the first 5000 turns. Figure 7 shows the growing amplitude of a bunch among the others in the same range.

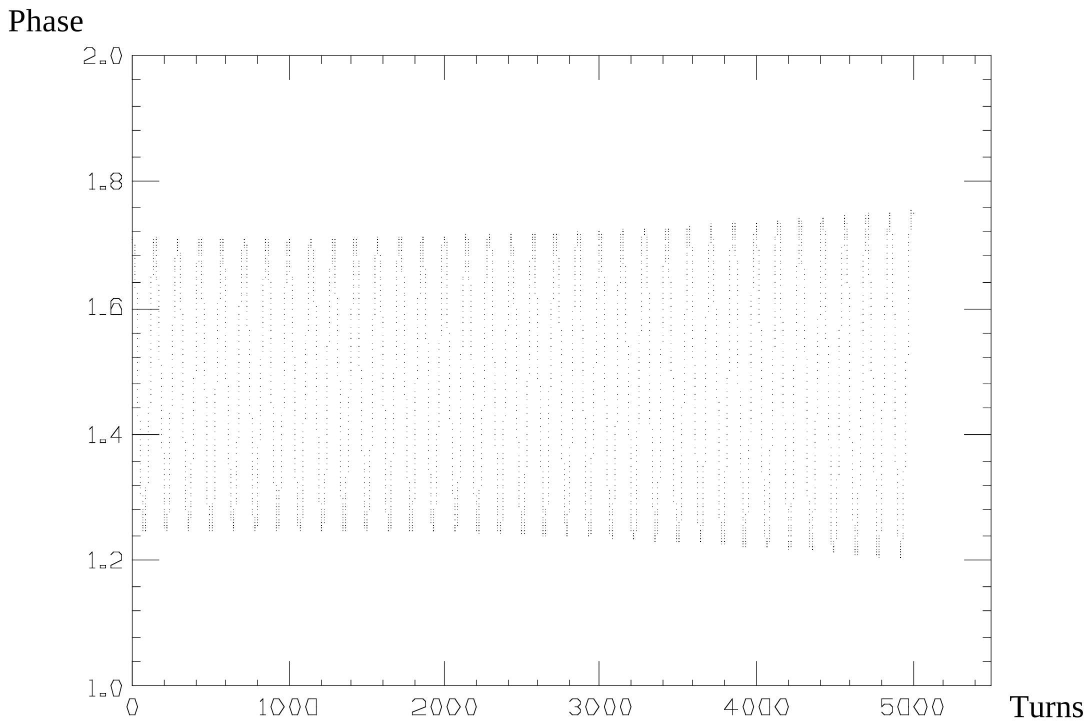

Fig. 6 - Phase oscillations of the injected bunch with the feedback off.

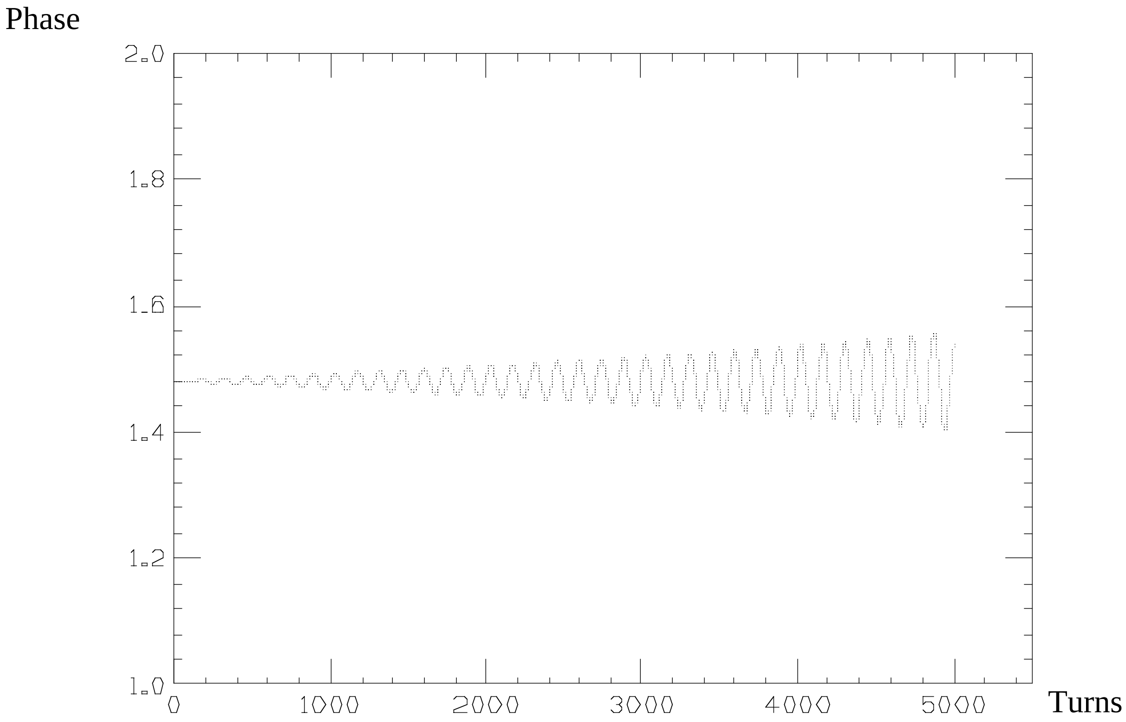

Fig. 7 - Phase oscillations of a bunch with the feedback off.

We have afterwards found a feedback configuration (Tab. 2) such to damp the oscillations with a kicker voltage of 400 Volt. Fig. 8 and Fig. 9 show the same bunches with the feedback on. In this case the whole system is stable.

Tab. 2 - Feedback parameters.

DOWN SAMPLING FACTOR	29
NUMBER OF FILTER COEFFICIENTS	5
FILTER COEFFICIENTS	3.5415E-01
	9.9797E-01
	2.3239E-01
	-8.6138E-01
	-7.3866E-01
PICK UP IMPEDANCE	1.0
ATTENUATOR + COMB COEFF.	1.0
AMPLIFIER GAIN	10.87
MIXER CONVERSION RATE	0.5
LOCAL OSCILLATOR GAIN	4.0
PHASE ERROR FOR L.O.	0.0
MATCHING AMPLIFIER GAIN	4.0
MAXIMUM INPUT VOLTAGE FOR ADC	.25
NUMBER OF BIT FOR ADC	8
PICKUP COUPLING FACTOR (%)	3.0
KICKER COUPLING FACTOR (%)	0.1
NOISE COEFFICIENT (%)	1.0
DSP GAIN	1.0
DAC GAIN	5.0
NUMBER OF BIT FOR DAC	8
MAX VOLTAGE FOR THE KICKER	400.0

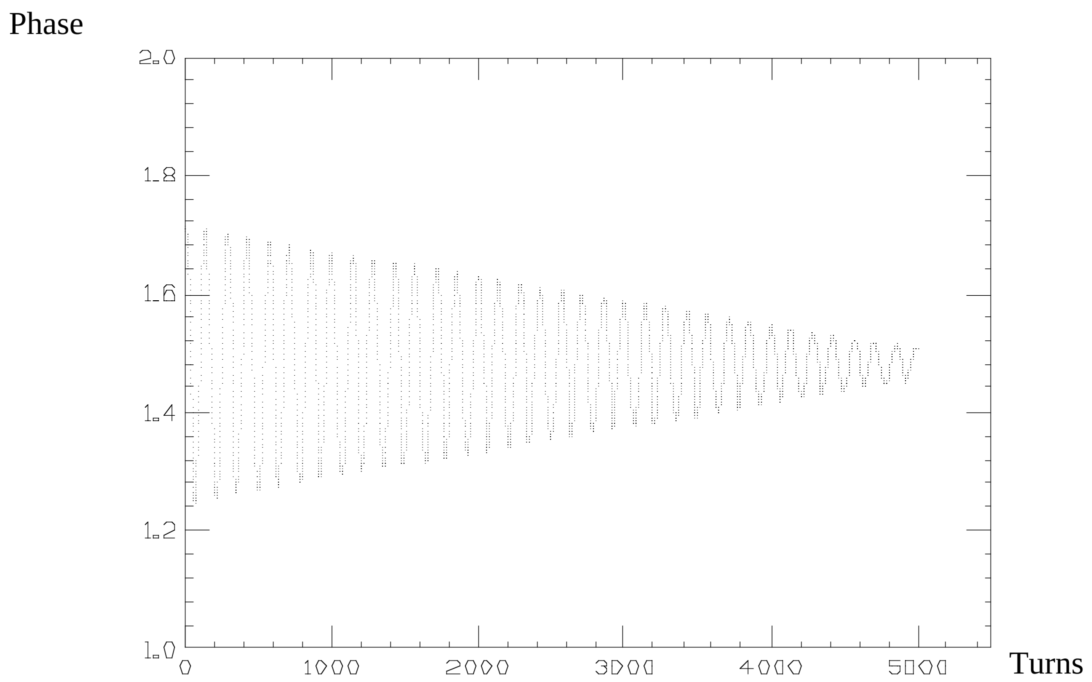

Fig. 8 - Phase oscillations of the injected bunch with the feedback on.

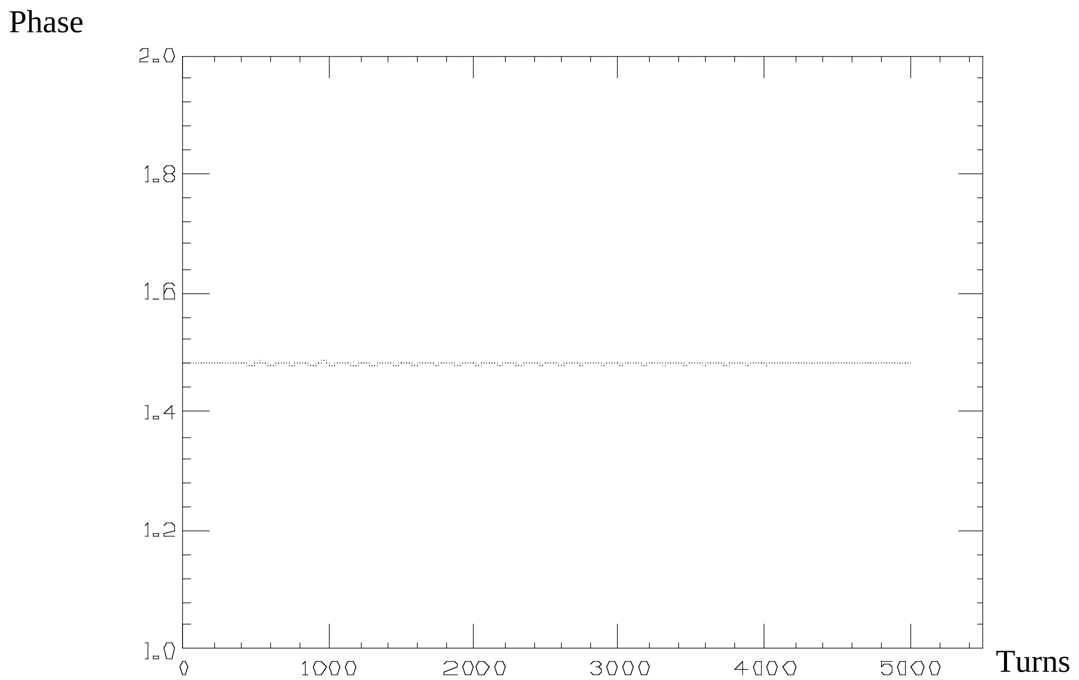

Fig. 9 - Phase oscillations of a bunch with the feedback on.

To be sure that the injection of the 30th bunch was the most dangerous for the stability point of view, with the same feedback parameters, we have simulated the injection of the nth bunch with n-1 bunches at their synchronous phases. We have then reported the maximum phase excursion of the bunch more perturbed by this injection versus the number of the bunch injected. As expected the oscillations becomes larger increasing the total current stored in the machine.

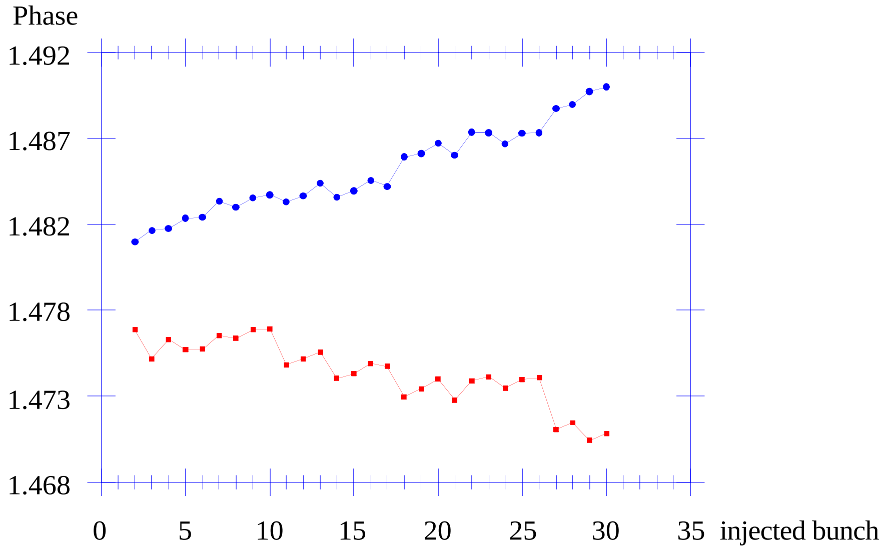

Fig. 10 - Maximum perturbation of the stored bunches.

Comparison of results obtained by a similar code developed at SLAC[7,8] shows some inconsistencies in the case of low quality factor Q of the HOMs. We presume that this is due to the approximations on the wake field expressions, valid only for high Q, adopted in the SLAC code.

5. CONCLUSIONS

The comparison between the simulation code and the theory results is satisfactory. We have found that with reasonable feedback parameters, even in the worst case of full coupling, it is possible to control the longitudinal dipole instabilities due to the HOMs, and to stabilize the beam. Furthermore we have now the possibility to simulate cases that could not be predicted analytically, and interesting for DAΦNE.

ACKNOWLEDGMENTS

We have had the possibility to utilize the SLAC simulation code [7,8]. We thank J. Fox, W. Hosseini, H. Hindi, K. Thompson for useful discussions on the program and I/O data structures.

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